CSI Acquisition Strategies for Interference Mitigation in Massive MIMO Networks

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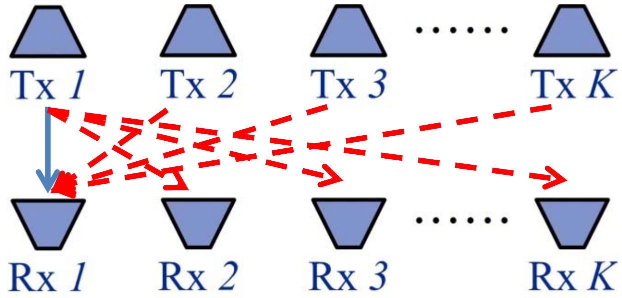
Motivation

Channel State Information (CSI) Acquisition

CSI Acquisition is Important to Achieve Interference Mitigation

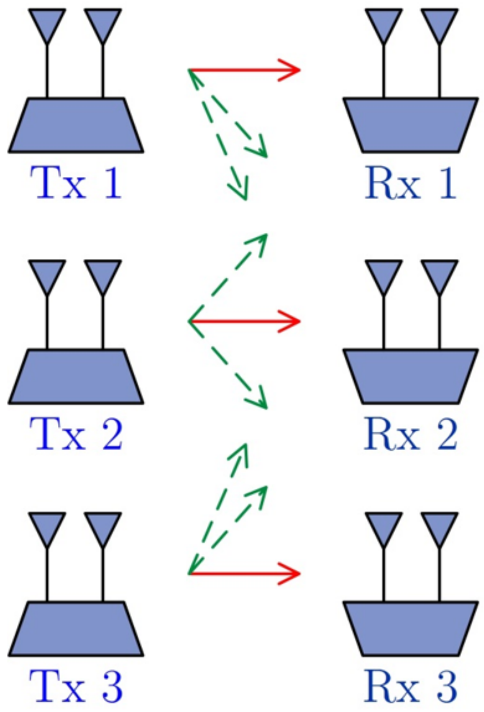
- **Interference** is a key performance bottleneck of wireless communication

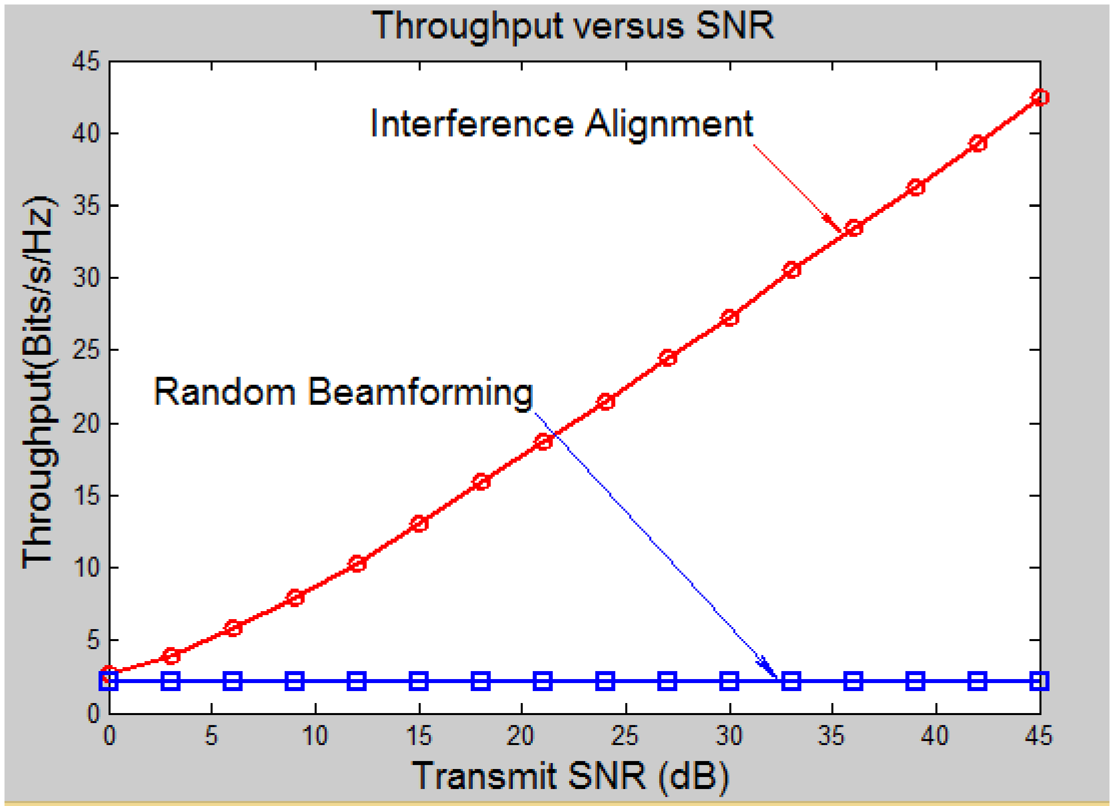


- By designing precoder / decorrelator *based on* CSI, **interference** can be effectively **mitigated** in many advanced techniques, e.g.,
  - Technique of zero forcing (ZF) [Jindal, *et.al.* 2006]
  - Technique of weighted MMSE (WMMSE) [Shi, Luo, *et.al.* 2011]
  - Technique of Interference alignment (IA) [Cadambe, Jafar, *et.al.* 2008]

Performance Gain of Interference Mitigation with CSI

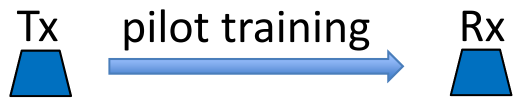
- And they also achieve dramatic performance gains:



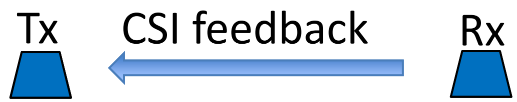


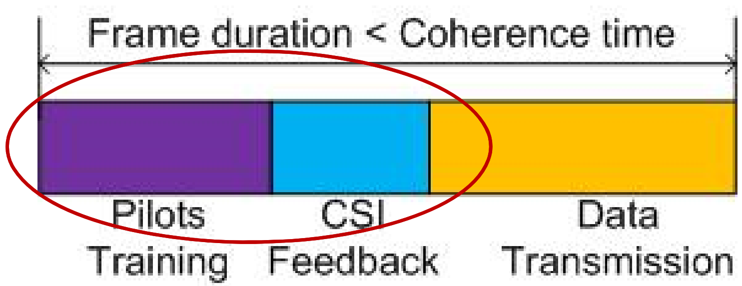
Acquiring CSI is Costly

- These interference mitigation techniques usually requires the CSIs at the transmitter (CSIT), which leads to a **heavy CSI acquisition overhead,**
  - E.g. how to acquire CSIT in FDD systems
    - Phase I: Transmitter (Tx) sends pilot training symbols to probe the channel on the forward channel



    - Phase II: Receiver (Rx) estimates and feedbacks the channel information to Tx side





Implication

- ✓ Reduce the CSI acquisition cost
- ✓ tradeoff analysis between the interference mitigation performance and the CSI acquisition cost.

Overview

Outline of My Thesis Research

Research Outline

CSI Acquisition Strategies

CSI feedback Reduction for IA

- in interference network with IA processing
  [**Rao**, Ruan, Lau, TSP 2013]
- in cellular network with IA processing
  [**Rao**, Lau, TSP 2014]

from MIMO

CSI Acquisition Design

- **in Point-to-point** massive MIMO
  [**Rao**, Lau, *submitted to* TSP 2014]
- In Multi-user massive MIMO
  [**Rao**, Lau, TSP 2014]

to **Massive** MIMO

Background

CSI Feedback Reduction for IA

Background of IA

- Technique of interference alignment (IA)
  - **Basic idea**: Align the interference from different Txs into a lower dimensional subspace at the Rxs
  - **Performance advantage**: Achieve optimal capacity scaling law w.r.t. SNR [Cadambe, Jafar, et.al. 2008]

IA in MIMO interference network

- Network Topology

1: K-user MIMO interference network
2: The i-th Tx and Rx have N_i and M_i antennas respectively
3: d_i data streams are demanded for the i-th Tx-Rx pair.

- Signal Model

The received signal $\mathbf{y}_j \in \mathbb{C}^{d_j \times 1}$ at the j-th Rx is given by

$$\mathbf{y}_j = \mathbf{U}_j^H \left(\sum_{i=1}^{K} \mathbf{H}_{ji} \mathbf{V}_i \mathbf{x}_i + \mathbf{n}_j \right)$$

- Problem Formulation

Problem 1.1 (Conventional IA). Find the set of precoders $\{\mathbf{V}_i \in \mathbb{C}^{N_i \times d_i} : \forall i\}$ and decorrelator $\{\mathbf{U}_j \in \mathbb{C}^{M_j \times d_j} : \forall j\}$, such that:

ise

al

$$\operatorname{rank}(\mathbf{U}_j^H \mathbf{H}_{jj} \mathbf{V}_j) = d_j, \forall j, \quad (1.1)$$

$$\mathbf{U}_j^H \mathbf{H}_{ji} \mathbf{V}_i = \mathbf{0}, \forall i, j, i \neq j. \quad (1.2)$$

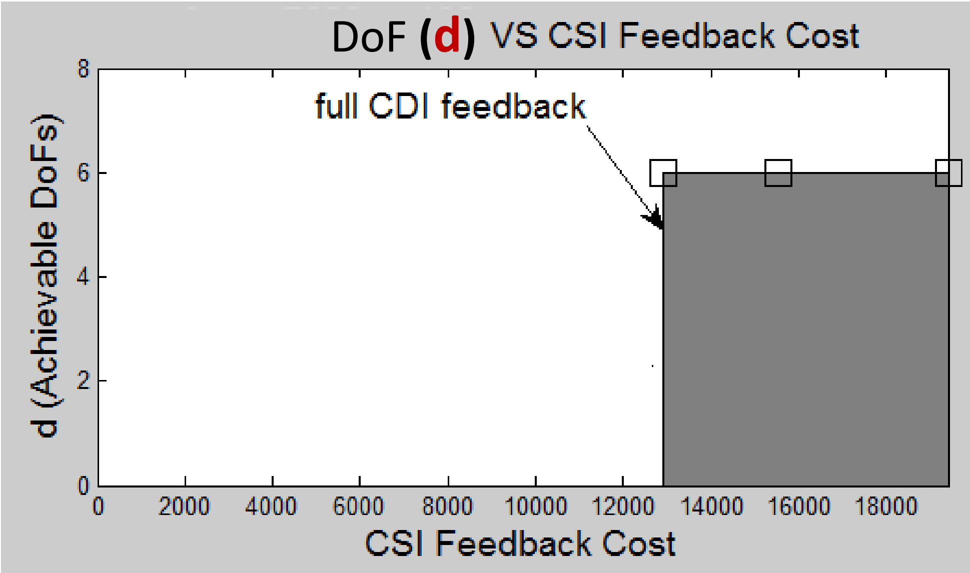
Conventional IA Results

- When Problem 1.1 feasible, $C = \left(\sum_{k=1}^{K} d_k\right) \log_2(\mathrm{SNR}) + o(\log_2(\mathrm{SNR}))$ → **DoF**
- There are two questions associated with Problem 1.1.
  - **Feasibility conditions**: when will Problem 1.1 has solution ?

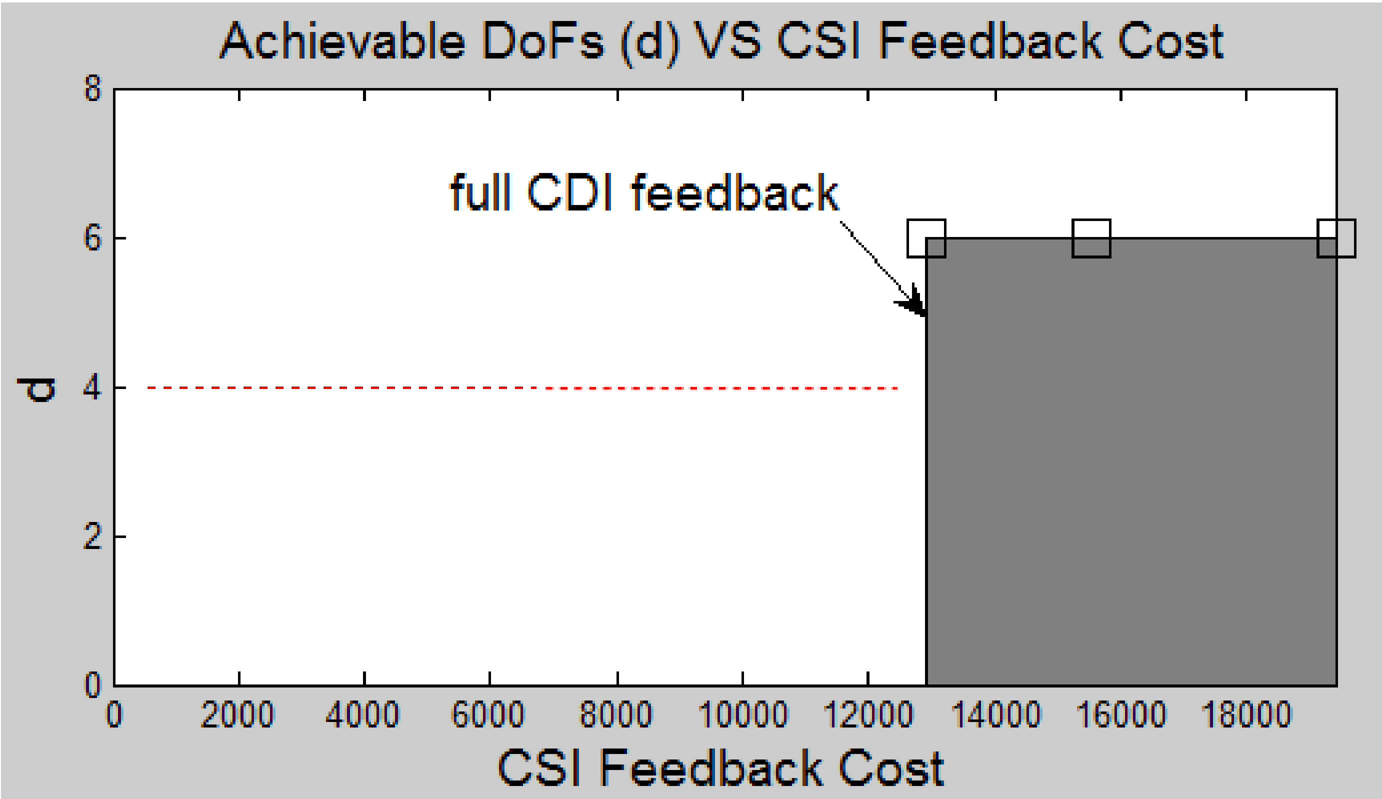
    (answer: feasibility study in [M. Razaviyanyn, Z. Luo, *et.al.* 2012])
  - **Transceiver design**: Given that Problem 1.1 is feasible, how to find the solution? (answer: AILM in [Gomadam, Jafar, *et.al.* 2011])
- Limitations of these IA designs: at least full channel direction information (CDI) is required.

Consider a $K = 6$ MIMO interference network where $M = 12$, $N = 36$ and $d_i = d$, $\forall i$.

Then from conventional IA study under full CDI, $d \leq 6$



Question: For a given data stream requirements $\{d_i : \forall i\}$ in the network, are there any approaches to **reduce the CSI feedback** overhead while still achieving IA?



Background of CSI Feedback Reduction

- Note that we focus on *CSI filtering*[1] only, which is different from CSI quantization ([Rajesh T. K. *et.al.* 2010]).

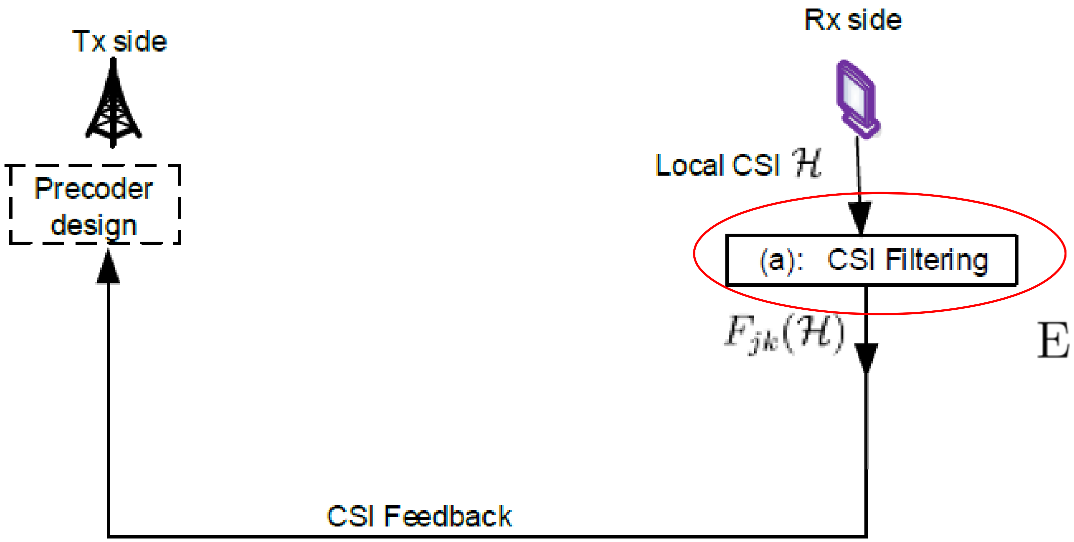


Figure 1.2: Illustration of CSI feedback reduction procedure.

E.g., for a (2×3) matrix $\mathbf{H}$,

1. $F(\mathbf{H}) = \mathbf{H}(1,:)$
2. $F(\mathbf{H}) = \{\mathbf{v} : \mathbf{Hv} = \mathbf{0}\}$

[1] In CSI filtering, only those parts of CSI that are essential to IA design are selected to be fed back.

Literature Works

- CSI feedback reduction for IA (focusing on CSI filtering)

Works	CSI feedback reduction	IA Design (Feasibility Conditions & Transceiver Algorithm)
[Razaviyan, et.al. 2012] [Jafar, et.al. 2011]	Full CSIT	Conventional
[K. R.T., et.al. 2010]	Full CDI feedback	Same as above
[Rezaee, et.al., 2013]	CSI submatrix feedback	Direct extension from above
This work	more holistic set of CSI reduction strategies	Different feasibility conditions and transceiver algorithms

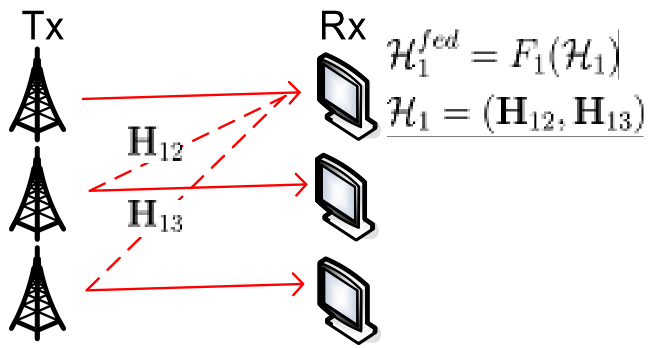
Table 1.2: A short summary of previous works on IA with partial CSIT.

System Model

- CSI Feedback Topology

  1: The Rx j has perfect local cross-link CSI $\mathcal{H}_j = (\cdots, \mathbf{H}_{ji}, \cdots)_{i \neq j}$.
  2: The Rx j feedbacks partial CSI $\mathcal{H}_j^{fed} = F_j(\mathcal{H}_j)$.
  3: The feedback CSI can be shared among the Txs.

- CSI Feedback Function $\mathcal{H}_j^{fed} = F_j(\mathcal{H}_j)$



The CSI feedback function at the j-th Rx F_j

$$F_j : \prod_{i=1}^{K} \mathbb{C}^{M_j \times N_i} \to \prod_{i=1}^{k_j} \mathbb{G}(A_{ji}, B_{ji})$$

The Grassmann manifold $\mathbb{G}(A, B)$, $(A \leq B)$, is the set of all A-dimensional linear subspaces in $\mathbb{C}^{B \times 1}$ $(A \leq B)$.

- Example

If $\mathbf{U}^H \mathbf{H} \mathbf{V} = \mathbf{0}$, then we have $\mathbf{U}^H (a\mathbf{H}) \mathbf{V} = 0$, $\forall a \in \mathbb{C}$. Hence, it is sufficient to feedback the *CDI* of $\mathbf{H} \in \mathbb{C}^{N \times M}$ for IA, i.e., $F(\mathbf{H}) = \{a\mathbf{H} : a \in \mathbb{C}\} \in \mathbb{G}(1, MN)$

CSI Feedback Cost

- How to quantify the CSI feedback cost associated with $\{F_j(\mathcal{H}_j)\}$

Define the CSI feedback cost as the sum dimension D of the Grassmann manifolds $\{\mathbb{G}(A_{ji}, B_{ji}) : \forall i, j\}$ that contain the CSI feedback function $\{F_j(\mathcal{H}_j)\}$, i.e.,

$$D = \sum_{j=1}^{K} \sum_{i=1}^{k_j} A_{ji}(B_{ji} - A_{ji}).$$

- First order metric of CSI feedback dimension to measure the CSI feedback cost

e.g., to keep a constant quantization distortion Δ, the quantization bits B has to scale with the dimension D as $B = \mathcal{O}\left(D \cdot \log \frac{1}{\Delta}\right)$.

Problem Formulation

- **Problem 1.2** (IA under Partial CSIT). Find $\{\mathbf{V}_j : \forall j\}$ and $\{\mathbf{U}_j : \forall j\}$, such that:

$$\text{rank}(\mathbf{U}_j^H \mathbf{H}_{jj} \mathbf{V}_j) = d_j, \quad \forall j, \tag{1.3}$$

$$\mathbf{U}_j^H \mathbf{H}_{ji} \mathbf{V}_i = 0, \quad \forall i, j, i \neq j \tag{1.4}$$

IA constraints

$$\{\mathbf{V}_i : \forall i\} \text{ can only be adaptive to the partial CSI feedback } \{F_j(\mathcal{H}_j) : \forall j\} \tag{1.5}$$

CSI knowledge constraint

- Three key questions associated with Problem 1.2

Q1: How to reduce CSI feedback for IA? i.e., what is {F_j}

Q2: IA feasible con

$$C = \left(\sum_{k=1}^{K} d_k\right) \log_2(\text{SNR}) + o(\log_2(\text{SNR}))$$

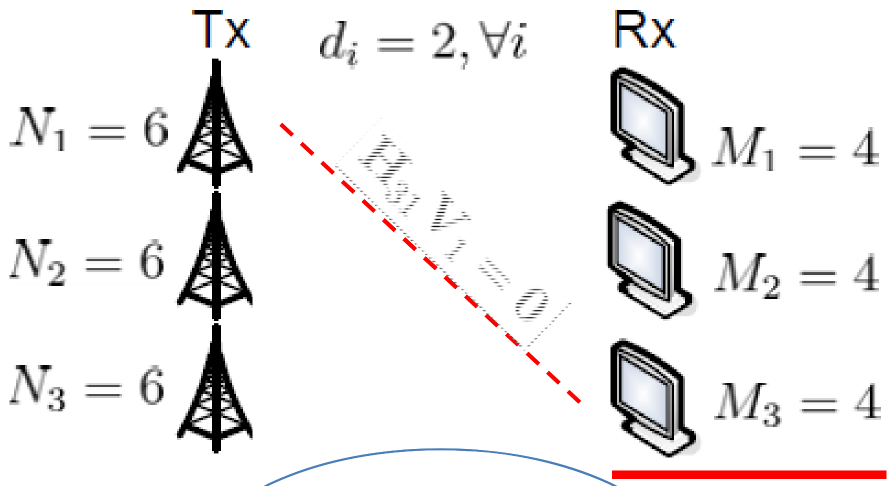
Q3: Find transceiver solution under partial CSIT

Proposed Solution to Q1

Q1: How to reduce CSI feedback for IA?, i.e., what is {F_j}

- **Toy Example I**

$$\mathbf{H}_{31}\mathbf{V}_1 = \mathbf{0}$$
$$\mathbf{H}_{12}\mathbf{V}_2 = \mathbf{0}$$
$$\mathbf{H}_{23}\mathbf{V}_3 = \mathbf{0}$$



$$F_3(\mathcal{H}_3) = \mathbb{N}^t(\mathbf{H}_{31})$$

(a)

– Insights:

- **Strategy I**: No feedback for a subset of cross links
- **Strategy II**: feedback of null space for a subset of cross links.

Full CDI	Proposed
138	24

Proposed Solution to Q1

Q1: How to reduce CSI feedback for IA?, i.e., what is {F_j}

- **Toy Example II:** Strategy III

Details of Example II

- **Toy Example III:** Strategy IV

Details of Example III

Proposed Solution to Q1

Q1: How to reduce CSI feedback for IA?, i.e., what is {F_j}

- CSI feedback scheme (characterized by $\mathcal{L}$)

Define the CSI feedback profile as

$$\mathcal{L} = \left\{ \{M_i^s, N_i^s : \forall i\}, \{\Omega_j^I, \Omega_j^{II}, \Omega_j^{III}, \Omega_j^{IV} : \forall j\} \right\}$$

where $\{M_j^s, N_i^s\}$ controls the size of the CSI submatrices to feedback and $\{\Omega_j^m : \forall m\}$ defines the partitioning of the cross links w.r.t. the four feedback strategies at the j-th Rx.

  - There is a 1-1 correspondence between the feedback profile $\mathcal{L}$ and feedback function $\{F_j\}$

Detailed expression of F according to L

- The associated feedback cost is given by $D(\mathcal{L})$ (a function of $\mathcal{L}$)

Proposed Solution to Q2

Q2: IA feasible conditions under partial CSIT

- We then obtain ***sufficient conditions*** on $\mathcal{L}$ to ensure that **Problem 1.2** has solutions, i.e.,

$$\mathcal{L} \in \mathbb{L}_{sf}$$

Details of feasibility conditions

Proposed Solution to Q3

Q3: Find transceiver solution under partial CSIT

- We propose a novel transceiver algorithm under the proposed CSI feedback scheme

The proposed modified AILM solves the following two problems $(\mathcal{P}_1, \mathcal{P}_2)$ alternatively until convergence.

$$\mathcal{P}_1 : \min_{\{\mathbf{U}_j^b : (\mathbf{U}_j^b)^H \mathbf{U}_j^b = \mathbf{I}_{d_j^0}, \forall j\}} I = \sum_{j,i:\, i \in \Omega_j^{IV}} \mathrm{tr}\left(\left((\mathbf{U}_j^b)^H \mathbf{G}_{ji} \mathbf{V}_i^a \right) \left((\mathbf{U}_j^b)^H \mathbf{G}_{ji} \mathbf{V}_i^a \right)^H \right).$$

$$\mathcal{P}_2 : \min_{\{\mathbf{V}_i^a \in \mathbb{C}^{N_i^e \times d_i} : (\mathbf{V}_j^a)^H \mathbf{V}_j^a = \mathbf{I}_{d_j}, \forall i\}} I = \sum_{j,i:\, i \in \Omega_j^{IV}} \mathrm{tr}\left(\left((\mathbf{U}_j^b)^H \mathbf{G}_{ji} \mathbf{V}_i^a \right) \left((\mathbf{U}_j^b)^H \mathbf{G}_{ji} \mathbf{V}_i^a \right)^H \right).$$

With the converged solution $\{\mathbf{V}_i^a, \mathbf{U}_j^b\}$, the overall solution to Problem 1.2 is given by

$$\mathbf{V}_i = \begin{bmatrix} \mathbf{S}_i^t \mathbf{V}_i^a \\ \mathbf{0} \end{bmatrix}, \quad \mathbf{U}_j = v_{d_j} \left(\sum_{i \neq j} (\mathbf{H}_{ji} \mathbf{V}_i) (\mathbf{H}_{ji} \mathbf{V}_i)^H \right), \forall i, j$$

A short summary of the results

- Problem of IA under Partial CSIT

Q1: How to reduce CSI feedback for IA?

A1: We propose a CSI feedback profile which contains several control parameters $\mathcal{L}$

Q2: IA feasible condition under partial CSIT

A2: We establish sufficient conditions $\mathcal{L} \in \mathbb{L}_{sf}$

Q3: IA transceiver design under partial CSIT

A3: Implementation of IA under partial CSI (i.e., how to find the transceivers to satisfy the IA conditions)

CSI Feedback Profile Design $\mathcal{L}$

- Problem formulation

The problem of reducing the CSI feedback overhead subject to a given data steams requirement for the Tx-Rx pairs $\{d_i : \forall i\}$, can be formulated as :

Problem 1.3 (CSI Feedback Design).

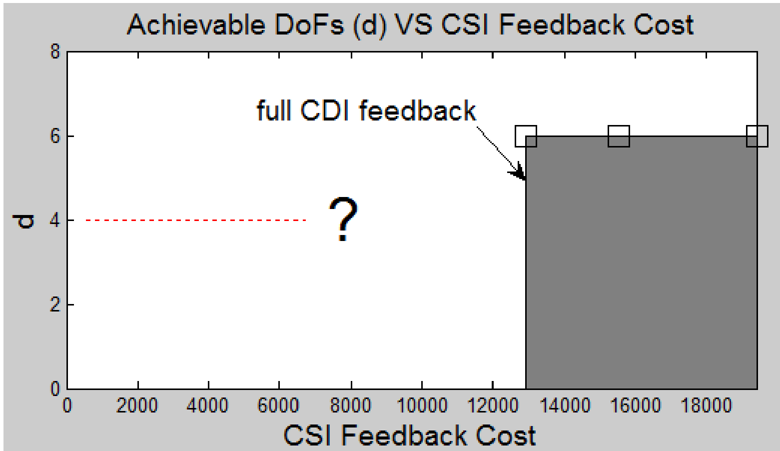
$$\min_{\mathcal{L}} \quad D(\mathcal{L})$$
$$\text{s.t.} \quad \mathcal{L} \in \mathbb{L}_{sf}.$$

Min {CSI feedback Cost}

DoFs achieved

1: Problem 1.3 is very challenging due the combinatorial nature of CSI feedback profile designs ($\mathcal{L}$).

2: We propose a low-complexity greedy algorithm to derive solutions.



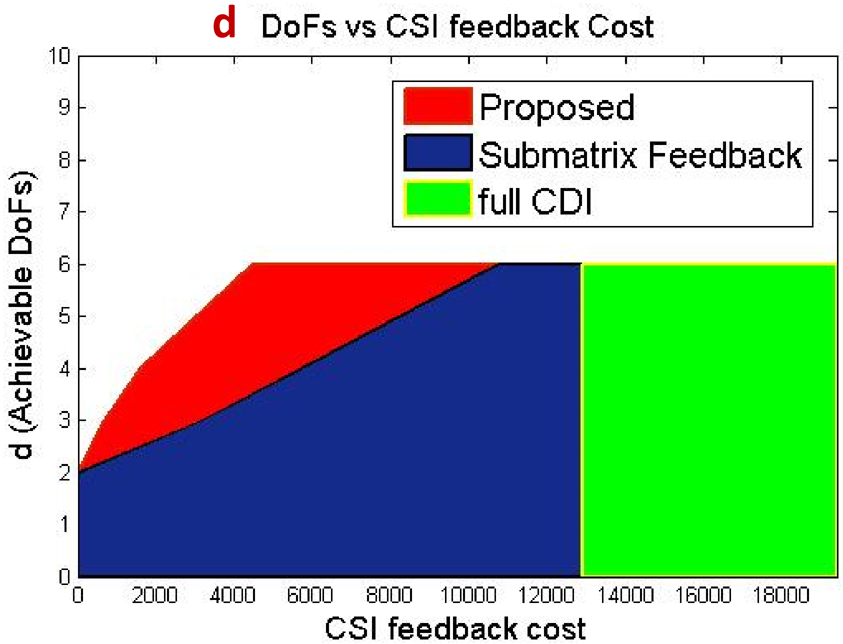
Performance-cost Tradeoff Results

Theorem 1.2 (Performance-Cost Tradeoff on a Symmetric MIMO Interference Network). *Consider a K-user MIMO interference network where $d_i = d$, $M_i = M$, $N_i = N$, $\forall i$ and M, N, d satisfy $2 \mid M$, $M \leq 2K+1$, $N = \frac{1}{2}KM$, $d \mid M$. The tradeoff between the data stream d and the feedback dimension D_p is summarized below:*

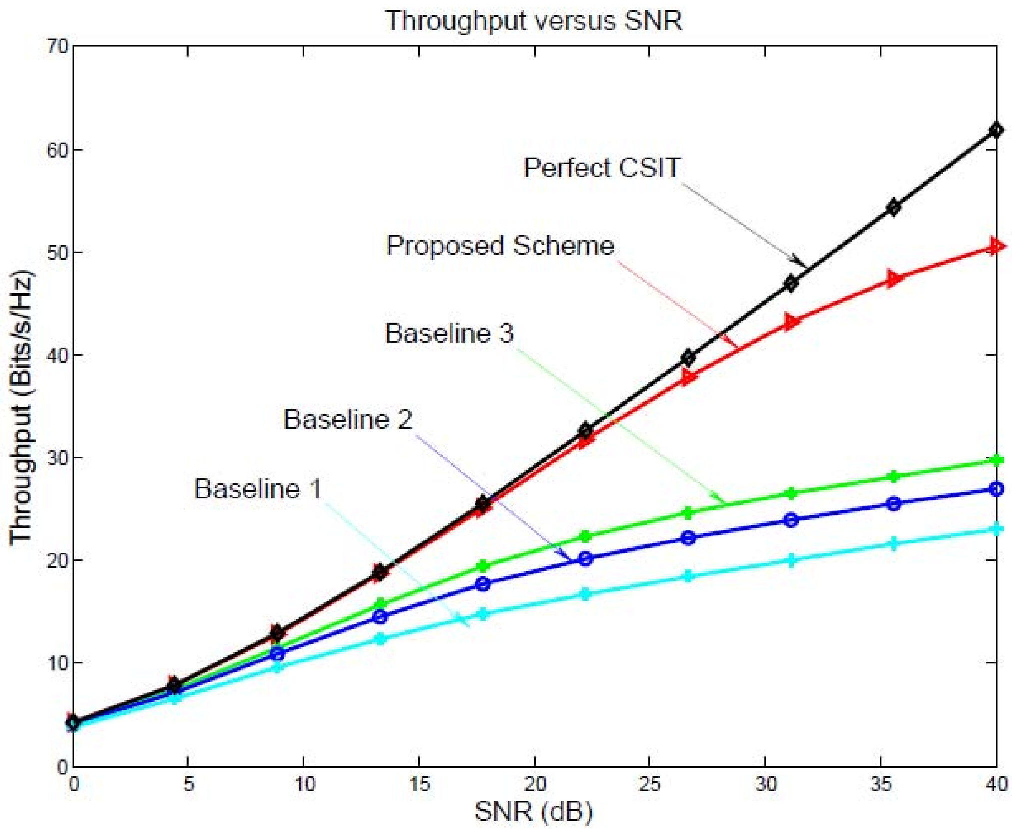
Data Stream d	*Feedback dimension D_p*	*full CDI D_{full}* [K. R.T., *et.al.* 2010]	*Submatrix Feedback D_s* [Rezaee, *et.al.*, 2013]
$d \leq \frac{M}{K}, d \mid M$	0	$K(K-1) \cdot (\frac{1}{2}KM^2 - 1)$	$K(K-1) \cdot (M \times ((K+1)d - M) - 1)^+$
$d = \frac{M}{K-\kappa}$, $1 \leq \kappa \leq K-2$	$((K+1)d^2 - Md) \cdot (K-1)^2$		

Consider a $K = 6$ MIMO interference network where $M = 12$, $N = 36$ and $d_i = d$, $\forall i$.

Then from conventional IA feasibility study under full CDI, $d \leq 6$



Simulation Results



Baseline1:
full CDI

Baseline 2:
Truncated feedback

Baseline 3:
CSI submatrix feedback

Figure 3.4: Throughput versus SNR under a $K = 4$, $[N_1, \cdots N_4] = [5, 4, 4, 3]$, $[M_1, \cdots M_4] = [4, 3, 2, 4]$, $[d_1, \cdots d_4] = [2, 1, 1, 1]$ MIMO interference network and the sum feedback bit constraint is 400.

(Thesis Chapter 4) Extension to

MIMO Cellular Networks

System Model

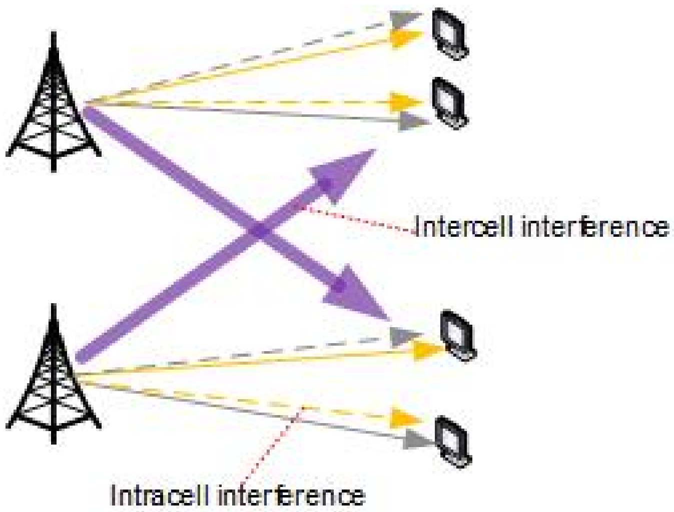
- MIMO Cellular Network
  - G base stations (BSs) and each BS serves K mobile station (MS)
  - Each BS and MS have N and M antennas, respectively.
  - *d* data streams are transmitted from the BS to each MS (Given DoF requirement).
- CSI Feedback

k-th MS of BS j

1. The (j,k)-th MS has perfect local CSI $\mathcal{H}_{jk} = (\mathbf{H}_{jk,1}, \mathbf{H}_{jk,2}, \cdots \mathbf{H}_{jk,G})$
2. The (j,k)-th MS feedbacks partial CSI $\mathcal{H}_{jk}^{fed} = F_{jk}(\mathcal{H}_{jk})$
3. The feedback CSI can be shared among the BSs $\{1, .., G\}$.

- Challenge

There are both intra-cell and inter-cell interference in cellular networks, which makes the dependencies of IA precoders on CSIs, very complicated.



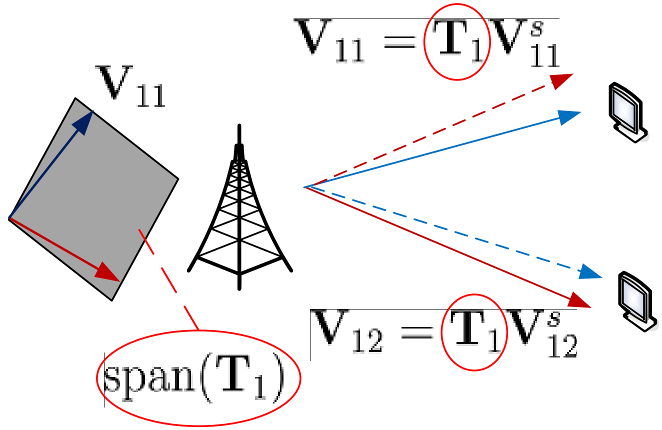
Problem Formulation

- Two Stage Precoding at BS

The precoder for the (j,k)-th MS is given by

$$\mathbf{V}_{jk} = \mathbf{T}_j \mathbf{V}_{jk}^s$$

$\mathbf{T}_j \in \mathbb{C}^{N \times Kd}$: Outer precoder at j-th BS.
$\mathbf{V}_{jk}^s \in \mathbb{C}^{Kd \times d}$: Inner precoder for the (j,k)-th MS.



- Problem formulation

Problem 2.1 (IA under Partial CSIT). Find $\left\{\mathbf{T}_j, \mathbf{V}_{jk}^s\right\}$ and $\{\mathbf{U}_{jk}\}$, such that

$$\mathrm{rank}(\mathbf{U}_{jk}^H \mathbf{H}_{jk,j} \mathbf{T}_j \mathbf{V}_{jk}^s) = d, \forall j, k;$$

$$\mathbf{U}_{jk}^H \mathbf{H}_{jk,j} \mathbf{T}_j \mathbf{V}_{jp}^s = \mathbf{0}, \forall j, k \neq p; \quad \text{(intracell IA constraints)}$$

$$\mathbf{U}_{jk}^H \mathbf{H}_{jk,i} \mathbf{T}_i = \mathbf{0}, \forall j, k, i \neq j; \quad \text{(intercell IA constraints)}$$

IA constraints

$\left\{\mathbf{T}_j, \mathbf{V}_{jk}^s : \forall j, k\right\}$ can only be adaptive to partial CSI $\{F_{jk}(\mathcal{H}_{jk}) : \forall j, k\}$. (CSI knowledge constraint)

CSI knowledge constraint

Proposed Scheme

- Regarding Problem 2.1, we shall answer the following questions:

Q1: How to reduce CSI feedback for IA?, i.e., what is {F_jk}

A1: CSI feedback profile $\mathcal{L}$. with CSI feedback cost $D(\mathcal{L})$

Q2: IA feasible conditions under partial CSIT

A2: Necessary Conditions

$\mathcal{L} \in \mathbb{L}_{ne}^{'}$

Sufficient Conditions

$\mathcal{L} \in \mathbb{L}_{sf}^{'}$

Q3: How to find transceiver solution under partial CSIT

A3: A novel transceiver algorithm

----Extended from AILM

----Adapt to *partial CSI* in MIMO *cellular network*

CSI Feedback Profile Design $\mathcal{L}$

- Problem formulation
  - The problem of reducing the CSI feedback cost subject to a given *d* data steams requirement for each MS, can be formulated as :

Problem 2.2 (CSI Feedback Design).

$$\min_{\mathcal{L}} \quad D(\mathcal{L})$$
$$\text{s.t.} \quad \text{Problem 2.1 is feasible under } \mathcal{L}. \qquad (2.2)$$

$\mathcal{L} \in \mathbb{L}'_{sf}$ (Suff. cond)
Upper Bound

  - We replace (2.2) with its sufficient conditions $\mathcal{L} \in \mathbb{L}'_{sf}$, and propose a low complexity *achievable* solution $\mathcal{L}_0$ that satisfies the sufficient conditions

$$D(\mathcal{L}^*) \leq D(\mathcal{L}_0)$$

CSI Feedback Profile Design $\mathcal{L}$

Problem 2.2 (CSI Feedback Design).

$$\begin{aligned} \min_{\mathcal{L}} \quad & D(\mathcal{L}) \\ \text{s.t.} \quad & \text{Problem 2.1 is feasible under } \mathcal{L}. \end{aligned} \qquad (2.2)$$

$\mathcal{L} \in \mathbb{L}_{ne}^{'}$ (Nece. cond)
Lower Bound

– We replace (2.2) with its necessary conditions $\mathcal{L} \in \mathbb{L}_{ne}^{'}$ and derive an lower bound on the optimal feedback dimension $D_{low} \leq D(\mathcal{L}^*)$

$$D_{low} \leq D(\mathcal{L}^*) \leq D(\mathcal{L}_0)$$

– We show that $D_{low} \rightarrow D(\mathcal{L}_0)$ as $G \rightarrow \infty$, and hence the proposed solution $\mathcal{L}_0$ is asymptotically optimal.

Corollary 2.1 (Asymptotic Optimality of $\mathcal{L}_0$). *Suppose the number of antennas N, M are given by $N = \lfloor C_1 K G \rfloor$, $M = \lfloor C_2 K G \rfloor$, where $0 < C_1, C_2 < d$, $d < C_1 + C_2$. As $G \rightarrow \infty$, we have*

$$\lim_{G\rightarrow\infty} \frac{D(\mathcal{L}_{low})}{G^4 K^3} = \lim_{G\rightarrow\infty} \frac{D(\mathcal{L}_0)}{G^4 K^3} = \frac{(d - C_1)(d - C_2)^2}{C_1}.$$

Chapter Summary

- We extend the framework to achieve CSI feedback reduction for IA from MIMO interference networks to MIMO cellular networks.

- The framework consists of the following components:
  - A set of CSI feedback reduction strategies for MIMO cellular network with IA processing
  - IA feasibility study under partial CSIT
  - IA transceiver algorithm under partial CSIT

- We post the problem of CSI feedback cost minimization subject to a given DoFs requirement, and we propose a low-complexity asymptotically optimal solution.

From MIMO to Massive MIMO

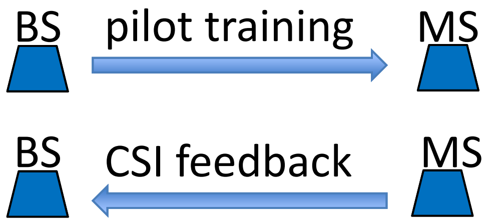
CSI Acquisition for Massive MIMO

Background

- **Benefits** of Massive MIMO (**BS** with **massive** antennas)

- **Challenge**: acquiring CSIT in massive MIMO is even more difficult

- Literature review for obtaining CSIT
  - Group I: Conventional (LS or MMSE) based CSIT estimation ([Biguesh, *et.al.* 2006]):
    - The training pilots from the BS scales as O(M), where M is the number of antennas at the BS



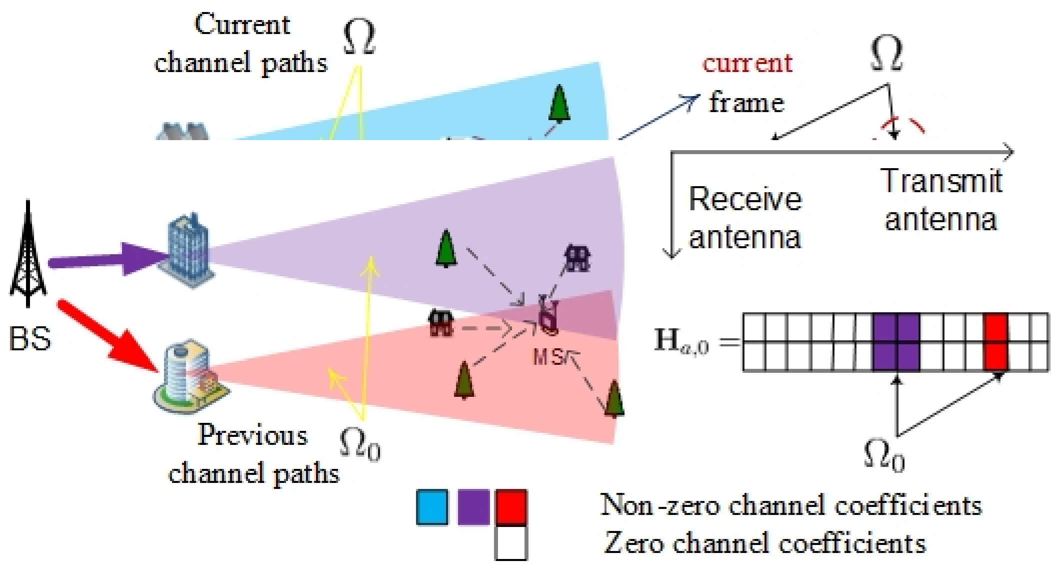
$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{N} \Longrightarrow$$

$$\hat{\mathbf{H}} = \mathbf{Y}\mathbf{X}^{\dagger}$$

  - Group II: Recent works based on technique of compressive sensing (CS) with exploitation of channel sparsity in massive MIMO ([Bajwa, *et.al.* 2010])
    - They consider *one-time static* channel estimation and ignore the correlation between channels across time.

Background

- *Massive MIMO Channels are jointly sparse and are temporally correlated.*
  - Sparse due to limited local scatters at BS (e.g., [Sayeed, *et.al.* 2006]).
  - Joint sparse due to the rich local scatterers at MS (e.g., [Hoydis, *et.al*. 2008]).
  - Temporally correlated due to the slowly varying propagation environment between the BS and MS.

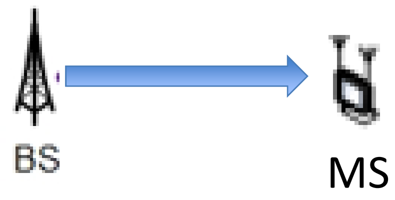


Target

*Exploit the **temporal correlation** and adapt to the **joint sparsity** to further enhance the channel estimation performance!*

System Model

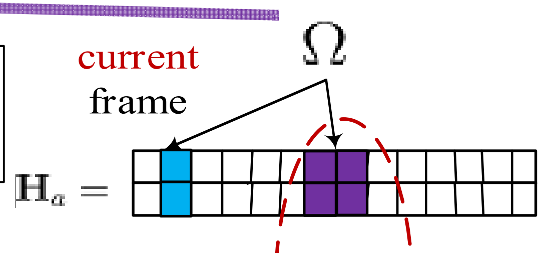
- Point-to-Point Massive MIMO
  - 1 BS with M (M is large) antennas and 1 MS with N antennas.



- Channel Model $\mathbf{H} = \mathbf{U}\mathbf{H}_a\mathbf{V}^H$

Massive MIMO Channel Model: Let $\mathbf{h}_j \in \mathbb{C}^{N\times 1}$ be the j-th row vector of $\mathbf{H}_a \in \mathbb{C}^{N\times M}$. The channel matrix $\mathbf{H}_a$ satisfies: $\text{supp}(\mathbf{h}_1) = \cdots \text{supp}(\mathbf{h}_N) \triangleq \Omega$, where Ω is the channel support and $|\Omega| \le \bar{s}$.

Statistic upper bound on the channel paths



- Modeling the Channel Temporal Correlation

Consider **Prior Channel Support Information** (Ω_0, s_c) available, where Ω_0 is the estimated channel support in the *previous* frame and s_c characterize the size of common channel paths between Ω_0 and Ω, i.e., $|\Omega_0 \bigcap \Omega| \ge s_c$.

Remark #: a larger s_c indicates a stronger temporal correlation between channels of consecutive frames

System Model

- Channel Estimation

Step 1: The BS sends training pilot $\mathbf{X} \in \mathbb{C}^{M \times T}$ in the downlink

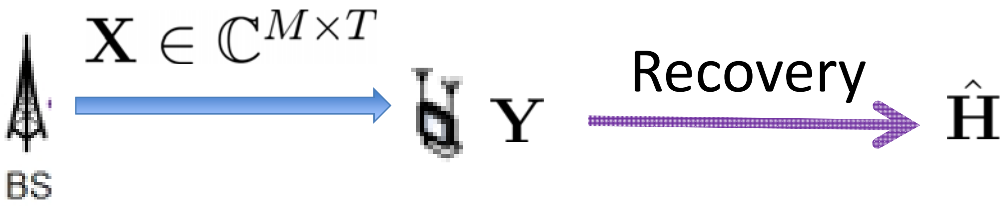
Step 2: The MS observes the channel output $\mathbf{Y} \in \mathbb{C}^{N \times T}$ given by

$$\mathbf{Y} = \sqrt{P}\mathbf{H}\mathbf{X} + \mathbf{N}$$

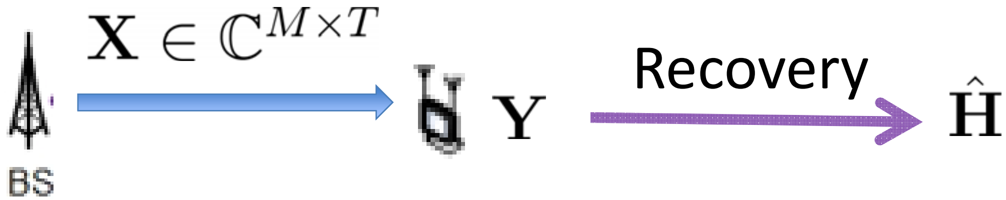
1: P transmit SNR

2: $\mathbf{N} \in \mathbb{C}^{N \times T}$ channel noise.

Step 3: The MS recovers the channel $\mathbf{H}$ based on the observed $\mathbf{Y}$.



T is the length of training pilots



Question

How to exploit prior channel support information (Ω_0, s_c),
How to adapt to the joint channel sparsity structure,
in the recovery to reduce the required length of training pilot T?

Proposed Scheme

- **Reformulation** as Compressive Sensing (CS) Signal Model

$$\underbrace{(\mathbf{Y}^H\mathbf{U})}_{\text{Measurements}} = \underbrace{\left(\sqrt{\frac{M}{T}}(\mathbf{V}\mathbf{X})^H\right)}_{\text{Measurement matrix}} \underbrace{\left(\sqrt{\frac{PT}{M}}(\mathbf{H}_a)^H\right)}_{\text{Unknown sparse source}} + \underbrace{(\mathbf{N}^H\mathbf{U})}_{\text{Noise}}$$

- Proposed Modified SP
  - Modified from conventional subspace pursuit (SP, [W. Dai, *et.al.* 2009])
  - Processing flow:

  - Design Features:
    - Incorporate prior support information (Ω_0, s_c) in the support identification.
    - Adapt to the joint sparsity of the source by identifying each row of $(\mathbf{H}_a)^H$ as an atomic unit.

Performance Analysis

- Performance Result

Theorem 3.1 (Channel Recovery Performance). *If* $\Phi = \sqrt{\frac{M}{T}}\mathbf{X}^H$ *satisfies a* s_2*-th order RIP with* $\delta_{s_2} \leq 0.153$*, where* $s_2 = 3\bar{s} + \min(0, |\Omega_0| - 3s_c)$*, then the recovered* $\hat{\mathbf{H}}$ *satisfies*

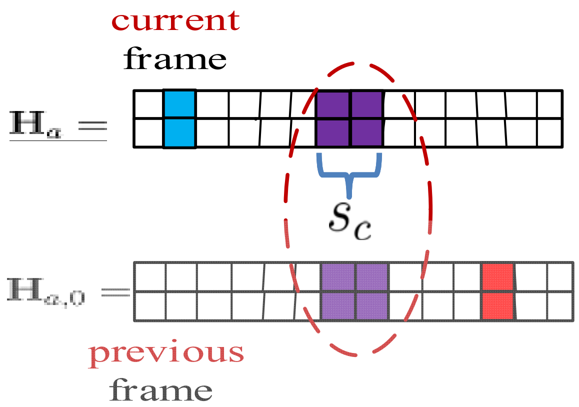
$$\mathbb{E}\left(\left\|\hat{\mathbf{H}} - \mathbf{H}\right\|_F\right) \leq \sqrt{\frac{M}{PT}}\left(\left(C_4 + \frac{1}{\sqrt{1-\delta_{s_2}}}\right)\frac{\Gamma\left(NT+\frac{1}{2}\right)}{\Gamma(NT)} + \frac{\gamma}{\sqrt{1-\delta_{s_2}}}\right), \quad (3.1)$$

where γ *is a threshold parameter in Algorithm 1,* C_4 *is a constant and depends on* δ_{s_2}*.*

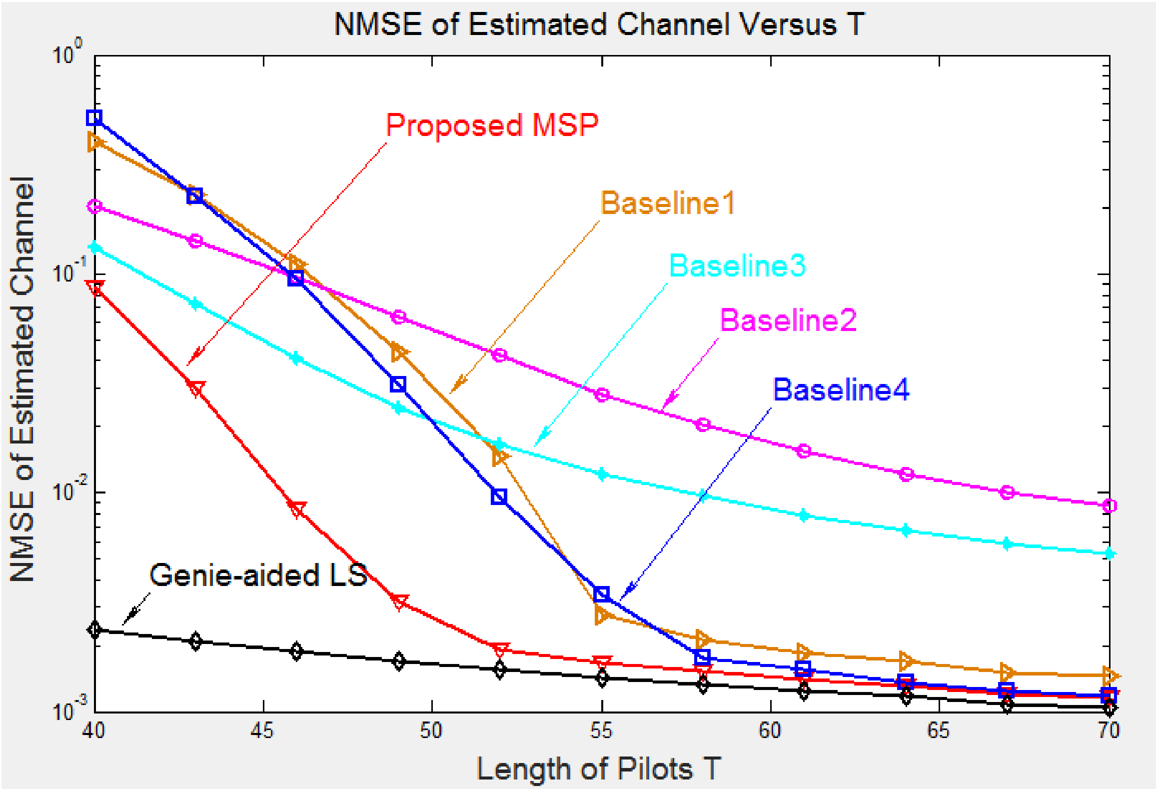
As the transmit SNR $P \to \infty$, $\left\|\hat{\mathbf{H}} - \mathbf{H}\right\|_F \to 0$.

- Implications
  - A stronger **temporal correlation** (a larger s_c) leads to a smaller requirement of number of training pilots T.

- The number of pilots T to achieve the s_2-th order RIP with $\delta_{s_2} = \delta$, is given by $T = C \cdot s_2 \log M$.
- $s_2 \triangleq 3\bar{s} + \min(0, |\Omega_0| - 3s_c)$ is monotonically decreasing as s_c increases, $\frac{|\Omega_0|}{3} \leq s_c \leq |\Omega_0|$.



Simulation Results



$$M = 200, N = 2, \bar{s} = 18,\ s_c = 10,\ P = 25\text{dB}$$

- ✓ Baseline1

  SP [W. Dai, *et.al.* 2009]
- ✓ Baseline2

  BP [Candes, *et.al.* 2005]
- ✓ Baseline3

  Modified-BP [Vaswani, *et.al.* 2010]
- ✓ Baseline4

  MMV-SP: modified from SP [W. Dai, *et.al.* 2009]

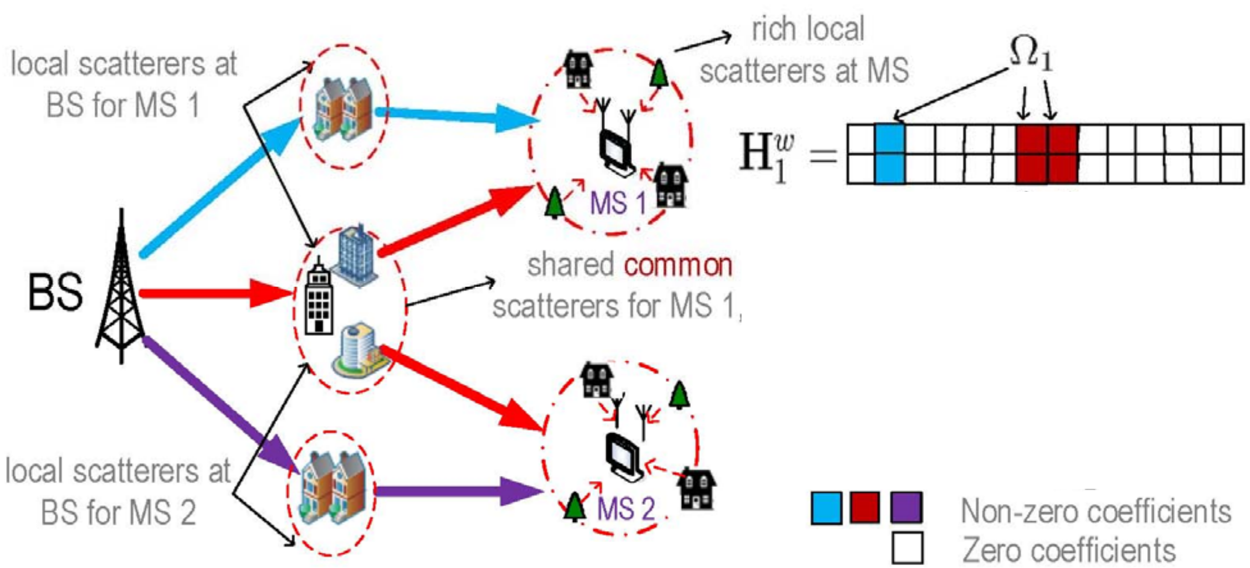
Summary

- CSI Acquisition in **Point-to-Point** Massive MIMO
  - We model the temporal correlation by considering a piece of prior channel support information available.
  - We then propose a novel scheme to enhance the channel estimation performance by exploiting the prior channel support information and adapting to the joint channel sparsity structure
  - We obtain simple results showing that the a *stronger temporal correlation* can achieve a better channel estimation performance.

CS-enabled CSIT Acquisition Design in Multi-user Massive MIMO

Background

- *Multi-user* massive MIMO channels are jointly sparse
  - Sparsity due to limited local scatters at BS (e.g., measurement report [Sayeed, *et.al.* 2006])
  - Structured sparsity (joint) due to some physical features:
    - Rich local scatterers at MS
    - Shared local scatterers between different MS (e.g., measurement report [Hoydis, *et.al.*, 2008])

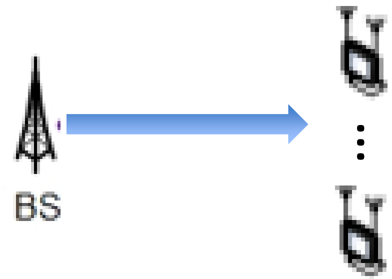


Target

Exploit not only the sparsity but also the joint sparsity among the user channels to further enhance the CSI acquisition efficiency!

System Model

- Network Topology
  - 1 BS (M antennas, M is large) and K MSs (N antennas).



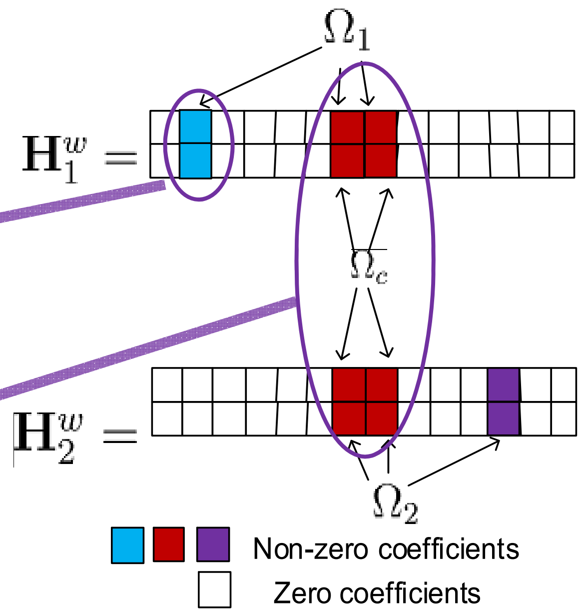
- Channel Model

(a) Individual joint sparsity due to local scattering at the BS: Denote $\mathbf{h}_{ij}$ as the j-th row vector of $\mathbf{H}_i^w$; then $\{\mathbf{h}_{ij} : \forall j\}$ are simultaneously sparse, i.e., there exists an index set Ω_i, $0 < |\Omega_i| \ll M$, $\forall i$, such that

$$\mathrm{supp}(\mathbf{h}_{i1}) = \mathrm{supp}(\mathbf{h}_{i2}) = \cdots \mathrm{supp}(\mathbf{h}_{iN}) \triangleq \Omega_i. \quad (4)$$

(b) Distributed joint sparsity due to common scattering at the BS: Different $\{\mathbf{H}_i^w : \forall i\}$ share a common support[3], i.e., there exists an index set Ω_c such that

$$\bigcap_{i=1}^{K} \Omega_i = \Omega_c. \quad (5)$$



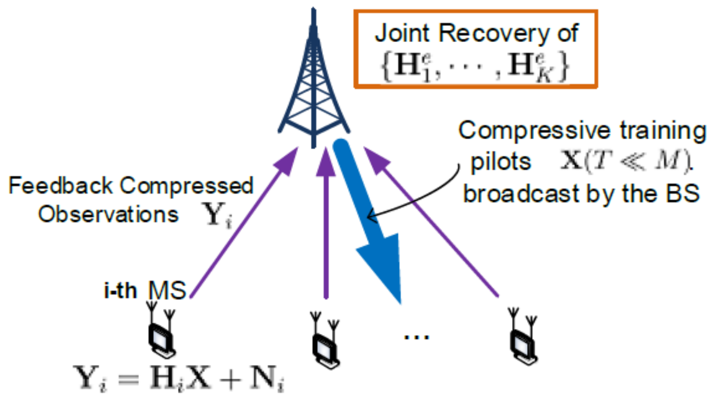
- Assumption of Statistical Sparsity Information

There exists statistical bound $\mathbb{S} = \{s_c, \{s_i : \forall i\}\}$ (s_c, $s_i \ll M$) available at the BS where $\Pr(\Lambda) > 1 - \varepsilon$ for some small ε and Λ denotes $|\Omega_c| \geq s_c$, $|\Omega_i| \leq s_i, \forall i$.

System Model

Algorithm 1 (Distributed Compressive CSIT Estimation and Feedback)

- **Step 1** *(Pilot Training)*: The BS sends the training symbols $\mathbf{X} \in \mathbb{C}^{M \times T}$.
- **Step 2** *(Channel Output Observation and Feedback)*: The i-th mobile user observes the output symbols $\mathbf{Y}_i$ given in (2) and feedbacks it to the BS side. $\mathbf{Y}_i = \mathbf{H}_i\mathbf{X} + \mathbf{N}_i, \forall i$
- **Step 3** *(Joint CSIT Recovery at BS)*: The BS recovers the CSIT $\{\mathbf{H}_i\}$ jointly based on the feedback $\{\mathbf{Y}_i : \forall i\}$. ■



1) Observation: the pilot training and feedback overheads both scale as the training length *O(T)*.

2) Reduce T by exploiting the joint channel sparsities in the user channel matrices

Question: How to jointly recover the channel based on compressed measurements $\{\mathbf{Y}_i\}$?

$$\mathbf{Y}_i = \mathbf{H}_i \mathbf{X} + \mathbf{N}_i, \forall i \xrightarrow{\text{Recovery}} \mathbf{H}_i^e, \forall i$$

Proposed Joint CSIT Recovery

- Proposed Joint-OMP Algorithm
  - **Extended** from conventional OMP to exploit the joint sparsity structures in the user channel matrices.
  - **Algorithm flow**

  - **Complexity analysis**: Consider $s_i = s, \forall i$ for simplicity, overall complexity of the J-OMP is $O(KMNsT)$

Question: What is the performance of the proposed CSIT estimation scheme

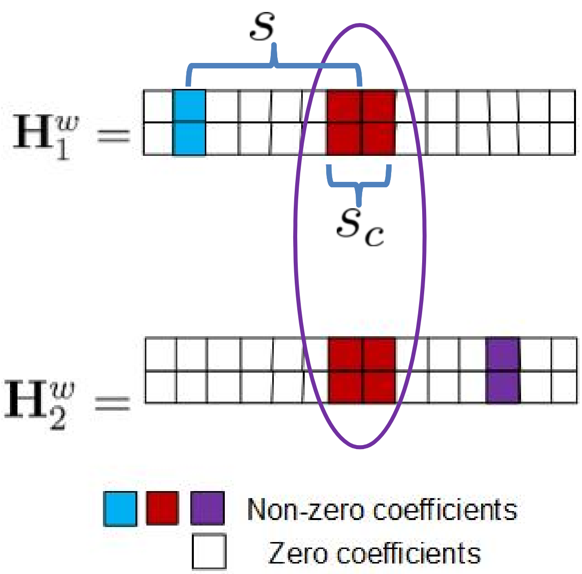
E.g., $$\mathbb{E}\left(\frac{\|\mathbf{H}_i - \mathbf{H}_i^e\|_F}{\|\mathbf{H}_i\|_F}\right)?$$

Insights

Corollary 5.3 (CSIT Quality w.r.t. Ω_c). *Suppose* $\varepsilon = 0$ *in (5.2.6). Scale the threshold parameter* η_2 *in Algorithm 5.2 as* $\eta_2 = \sqrt{P}$ *and let the transmit SNR* $P \to \infty$, *the number of users* $K \to \infty$. *If (5.3.8) holds and* p *in (5.3.9) satisfies* $p < \frac{1}{2}(1-\gamma)$, *we have*

$$\mathbb{E}\left(\frac{\|\mathbf{H}_i - \mathbf{H}_i^e\|_F}{\|\mathbf{H}_i\|_F}\right) \leq \left(\sum_{t=s_c}^{s} \binom{s}{t} - 1\right) E, \tag{5.4.8}$$

where $E = \left(\frac{1-\delta_s+\delta_{2s}}{1-\delta_s}\right) \times \left(\exp\left(-N\left(\ln\theta - 1 + \frac{1}{\theta}\right)\right) + M \cdot \exp\left(-N\left(\theta - 1 - \ln\theta\right)\right)\right).$



$$s_c \to s \Longrightarrow \mathbb{E}\left(\frac{\|\mathbf{H}_i - \mathbf{H}_i^e\|_F}{\|\mathbf{H}_i\|_F}\right) \to 0$$

Insights: a larger size of common support can benefit the CSIT estimation performance!

Chapter Summary

- CS-enabled scheme CSI acquisition scheme in ***multi-user*** FDD massive MIMO systems.
  - We build a model to incorporate the joint sparsity in the user channel matrices in multi-user massive MIMO system.
  - We propose a novel CS recovery algorithm (extended from conventional OMP) that can exploit the joint sparsity in the user channel matrices
  - We obtain simple analytical results showing that the joint channel sparsity can be captured by the proposed scheme to benefit the CSI estimation.

Overall Summary of Thesis Works

Overall Summary I

- CSI feedback reduction for IA in MIMO networks
  - **interference network topology**
  - **cellular networks topology**

Q1: How to reduce the CSI feedback for IA?

Q2: IA feasible condition under partial CSIT

Q3: IA transceiver design under partial CSIT

→

CSI feedback design

$$\min_{\mathcal{L}} D(\mathcal{L})$$
$$\text{s.t.} \, \mathcal{L} \in \mathbb{L}_{sf}$$

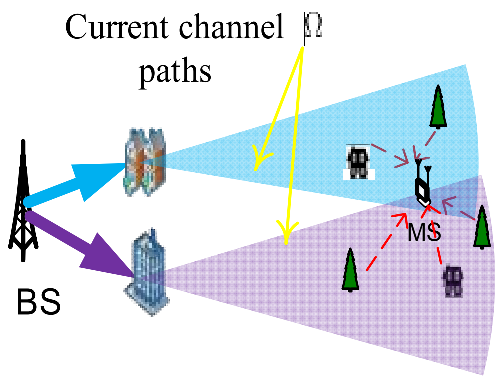
Overall Summary II

CSI Acquisition in massive MIMO

Point-to-Point Massive MIMO Systems

Feature: *Temporal correlations* in the multi-path profile

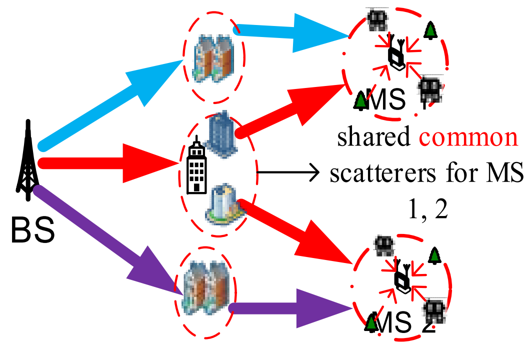
- Model the temporal correlation by considering a prior channel support and associated quality information available,
- Develop a novel scheme to exploit this prior information to enhance the channel recovery performance



Multi-user Massive MIMO Systems

Feature: *Joint channel Sparsity* among the users

- Build a model to incorporate the joint sparsity feature in the user channel matrices.
- Develop a novel CS-enabled scheme to exploit joint channel sparisty to enhance the channel estimation performance



Journal Publications

- Accepted / published
  1. X. Rao, V.K.N. Lau, "Minimization of CSI Feedback Dimension for Interference Alignment in MIMO Interference Multicast Networks," accepted for publication in ***IEEE Transactions on Information Theory***, Jan. 2015.
  2. X. Rao, V.K.N. Lau, "Distributed Fronthaul Compression and Joint Signal Recovery in Cloud-RAN,“ accepted for publication in ***IEEE Transactions on Signal Processing***, Dec, 2014.
  3. X. Rao, V.K.N. Lau, "Distributed Compressive CSIT Estimation and Feedback for FDD Multi-user Massive MIMO Systems," ***IEEE Transactions on Signal Processing***, vol. 62, no. 12, pp. 3261-3271, June 15, 2014.
  4. X. Rao, V.K.N. Lau, "Interference Alignment with Partial CSI Feedback in MIMO Cellular Networks," ***IEEE Transactions on Signal Processing***, vol.62, no.8, pp. 2100-2110, April 15, 2014.
  5. X. Rao, L. Ruan, V.K.N. Lau, "CSI Feedback Reduction for MIMO Interference Alignment," ***IEEE Transactions on Signal Processing***, vol. 61, no. 18, pp. 4428-4437, Sept. 15, 2013.
  6. X. Rao, L. Ruan, V.K.N. Lau, "Limited feedback design for interference alignment on MIMO interference networks with heterogeneous path loss and spatial correlations," ***IEEE Transactions on Signal Processing***, vol. 61, no. 10, pp. 2598-2607, May 15, 2013.
  7. L. Ruan, V.K.N. Lau, X.Rao, "Interference Alignment for Partially Connected MIMO Cellular Networks," ***IEEE Transactions on Signal Processing***, vol.60, no.7, pp. 3692-3701, July 2012.

- Submitted
  8. X. Rao, V.K.N. Lau, "Compressive Sensing with Prior Support Quality Information and Application to Massive MIMO Channel Estimation," ***submitted to IEEE Transactions on Signal Processing***,, Jan., 2015.

Thank you!

(*Any questions?*)

Backup slides

Overall Summary I

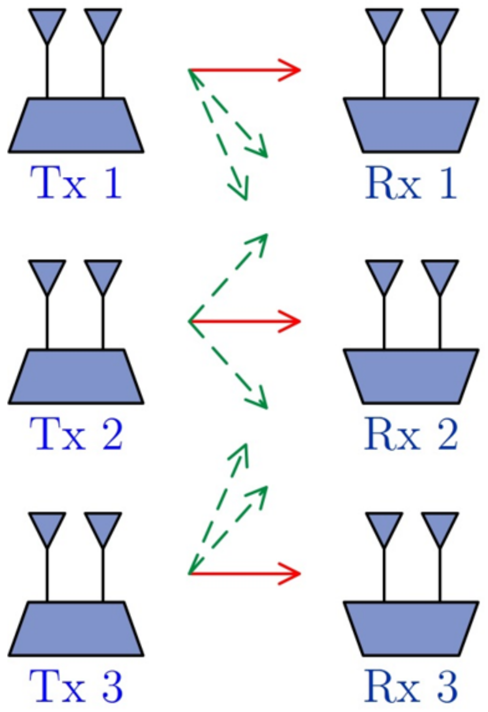
CSI feedback reduction for IA in MIMO networks

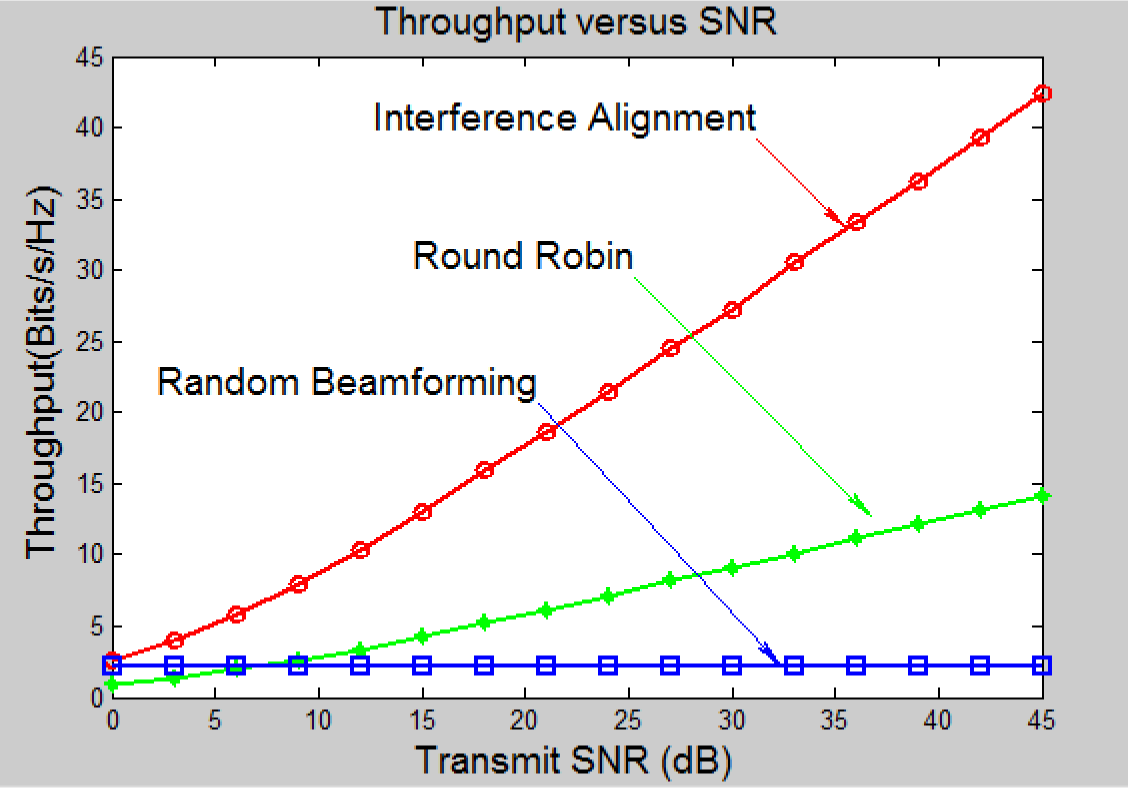
- Q1: How to reduce the CSI feedback for IA? → Define the considered strategy space $\mathbb{L}$ for $\mathcal{L}$, i.e., $\mathcal{L} \in \mathbb{L}$.
- Q2: IA feasible condition under partial CSIT → Define the constraints on $\mathcal{L} \in \mathbb{L}$ i.e., $\mathcal{L} \in \mathbb{L}_{sf} \subseteq \mathbb{L}$
- Q3: IA transceiver design under partial CSIT → Implementation: find $\{\mathbf{U}, \mathbf{V}\}$ to satisfy the conditions of IA under $\mathcal{L}$
- Q4: CSI feedback design → $\min_{\mathcal{L}} D(\mathcal{L})$ s.t. $\mathcal{L} \in \mathbb{L}_{sf}$

Backup slides for introduction

Performance Gain of Interference Mitigation with CSI

- **Comparison** of communication throughput with / without interference mitigation:





Backup slides for chapter 1

Proposed Solution to Q1

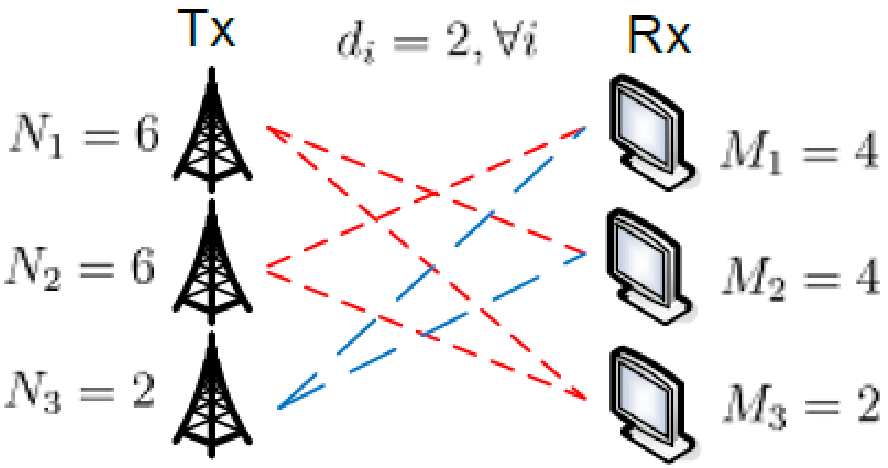
Q1: How to reduce CSI feedback for IA?, i.e., what is {F_j}

- **Toy Example II**

$$\mathrm{span}(\mathbf{V}_1) = \mathbb{N}^t\left((\mathbf{S}_2^r)^H\mathbf{H}_{21}\right) \bigcap \mathbb{N}^t(\mathbf{H}_{31})$$

$$\mathrm{span}(\mathbf{V}_2) = \mathbb{N}^t\left((\mathbf{S}_1^r)^H\mathbf{H}_{12}\right) \bigcap \mathbb{N}^t(\mathbf{H}_{32})$$

- **Strategy III**: Cancel part of the cross-links and feedback the aggregate CSI.



$$F_1(\mathcal{H}_1) = \mathbb{N}^t((\mathbf{S}_1^r)^H\mathbf{H}_{12})$$
$$F_2(\mathcal{H}_2) = \mathbb{N}^t\left((\mathbf{S}_2^r)^H\mathbf{H}_{21}\right)$$
$$F_3(\mathcal{H}_3) = (\mathbb{N}^t(\mathbf{H}_{32}), \mathbb{N}^t(\mathbf{H}_{31}))$$

(b)

Go back

Full CDI	Proposed
82	32

$$(\mathbf{S}_1^r)^H\mathbf{H}_{13} = \mathbf{0}$$
$$(\mathbf{S}_2^r)^H\mathbf{H}_{23} = \mathbf{0}$$

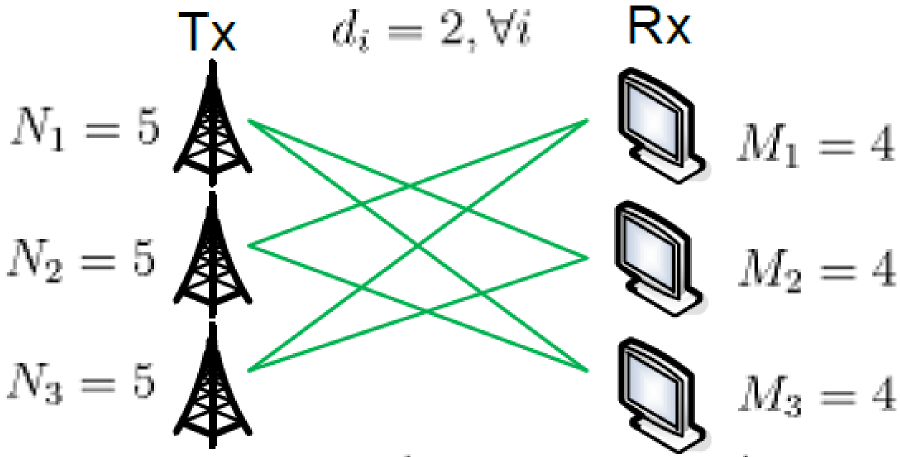
Proposed Solution to Q1

Q1: How to reduce CSI feedback for IA?, i.e., what is {F_j}

- **Toy Example III**

- **Strategy IV**: Feedback of Row Space of CSI Submatrices for a Subset of Cross Links

Full CDI	Proposed
114	48



$$F_1(\mathcal{H}_1) = \text{span}\left(\begin{bmatrix} \mathbf{H}_{12}^s & \mathbf{H}_{13}^s \end{bmatrix}^T\right)$$
$$F_2(\mathcal{H}_2) = \text{span}\left(\begin{bmatrix} \mathbf{H}_{21}^s & \mathbf{H}_{23}^s \end{bmatrix}^T\right)$$
$$F_3(\mathcal{H}_3) = \text{span}\left(\begin{bmatrix} \mathbf{H}_{31}^s & \mathbf{H}_{32}^s \end{bmatrix}^T\right)$$

(c)

$$\mathbf{H}_{ji}^s = \begin{bmatrix} \mathbf{I}_4 & \mathbf{0} \end{bmatrix} \mathbf{H}_{ji}$$

Go back

Expression of Feedback Function

There is 1-1 correspondence between the feedback profile $\mathcal{L}$ and the feedback function feedback function $\{F_j\}$. For a given $\mathcal{L}$, the CSI feedback function is given by

$$F_j(\mathcal{H}_j) = \left(\underbrace{\cdots, \mathbb{N}^t\left((\mathbf{S}_j^r)^H \mathbf{H}_{jp}^s\right), \cdots}_{\forall p \in \Omega_j^{III}}, \text{span}\left(\left[\underbrace{\cdots \quad (\mathbf{S}_j^r)^H \mathbf{H}_{jq}^s \quad \cdots}_{\forall q \in \Omega_j^{IV}} \right]^T \right) \right), \tag{1}$$

where $\mathbf{S}_j^r \in \mathbb{C}^{M_j^s \times M_j^e}$, $(\mathbf{S}_j^r)^H \mathbf{S}_j^r = \mathbf{I}_{M_j^e}$,

$$\text{span}(\mathbf{S}_j^r) = \mathbb{N}^r \left(\underbrace{\left[\cdots \quad \mathbf{H}_{ji}^s \quad \cdots \right]}_{\forall i \in \Omega_j^{II}} \right), \tag{2}$$

$$M_j^e = M_j^s - \sum_{i \in \Omega_j^{II}} N_i^s, \forall j. \tag{3}$$

Go back

$$\mathbf{H}_{ji}^s = \left[\begin{array}{cc} \mathbf{I}_{M_j^s} & \mathbf{0} \end{array} \right] \mathbf{H}_{ji} \left[\begin{array}{c} \mathbf{I}_{N_j^s} \\ \mathbf{0} \end{array} \right], \forall j, i. \tag{4}$$

Proposed Solution to Q2

Q2: IA feasible conditions under partial CSIT

- We obtain sufficient conditions on the CSI feedback profile $\mathcal{L}$ to ensure that Problem 1.2 has solutions.

Theorem 1.1 (Sufficient Feasibility Conditions). *If the feedback profile $\mathcal{L}$ satisfies the following conditions:*
1)$N_i^e = N_i^s - \sum_{j:\, i\in\Omega_j^{III}} M_j^e \geq d_i$, $M_j^e = M_j^s - \sum_{i\in\Omega_j^{II}} N_i^s \geq d_i^0\ \forall i$,
2) the row vectors of all the matrices $\{\mathbf{X}_{ji} : \forall j, i \in \Omega_j^{IV}\}$ are linearly independent, then ***Problem 1.2*** *(Proposed IA under Partial CSIT) is feasible almost surely, where*

$$\underbrace{\mathbf{X}_{ji}}_{d_i d_j^0 \times \overline{M}} = [\underbrace{\mathbf{0}}_{d_i d_j^0 \times m_{ji}} \quad (\mathbf{G}_{ji}^{(2)})^T \otimes \mathbf{I}_{d_j^0} \quad \underbrace{\mathbf{0}}_{d_i d_j^0 \times n_{ji}} \quad \mathbf{I}_{d_i} \otimes \mathbf{G}_{ji}^{(3)} \quad \underbrace{\mathbf{0}}_{d_i d_j^0 \times k_{ji}}] \qquad (1.10)$$

and $\overline{M} = \sum_{i=1}^K \left(d_i^0(M_i^e - d_i^0) + d_i(N_i^e - d_i)\right)$, $n_{ji} = \sum_{p=j+1}^K d_p^0(M_p^e - d_p^0) + \sum_{q=1}^{i-1} d_q(N_q^e - d_q)$, $m_{ji} = \sum_{p=1}^{j-1} d_p^0(M_p^e - d_p^0)$, $k_{ji} = \sum_{q=i+1}^K d_q(N_q^e - d_q)$, $d_j^0 = d_j + \sum_{i\in\Omega_j^I} d_i$.

Proposed Solution to Q2

Q2: IA feasible conditions under partial CSIT

- Explicit sufficient conditions on $\mathcal{L}$ under divisible cases.

Corollary 1.1 (Explicit Sufficient Feasibility Conditions). *Suppose* $d_i = d, \forall i$. *If the feedback profile* $\mathcal{L}$ *satisfies:*

1) $d \mid M_i^s,\ d \mid N_i^s,\ \forall i,$

2) $N_i^e = N_i^s - \sum_{j:\, i \in \Omega_j^{III}} M_j^e \geq d_i,\ M_j^s - \sum_{i \in \Omega_j^{II}} N_i^s \geq d_i^0\ \forall i,$

3) Denote $V_i = d_i(N_i^e - d_i),\ \forall i,\ U_j = d_j^0(M_j^e - d_j^0),\ \forall j;\ C_{ji} = d_j^0 d_i,$ *and* V_i, U_j *and* C_{ji} *satisfy*

$$\sum_{j:\,(j,i)\in\Omega_{sub}} U_i + \sum_{i:\,(j,i)\in\Omega_{sub}} V_i \geq \sum_{j,i:\,(j,i)\in\Omega_{sub}} C_{ji}, \quad \forall \Omega_{sub} \subseteq \{(j,i) : \forall j, i \in \Omega_j^{IV}\}. \quad (1.11)$$

then ***Problem 1.2*** *(Proposed IA under Partial CSIT) is feasible almost surely.*

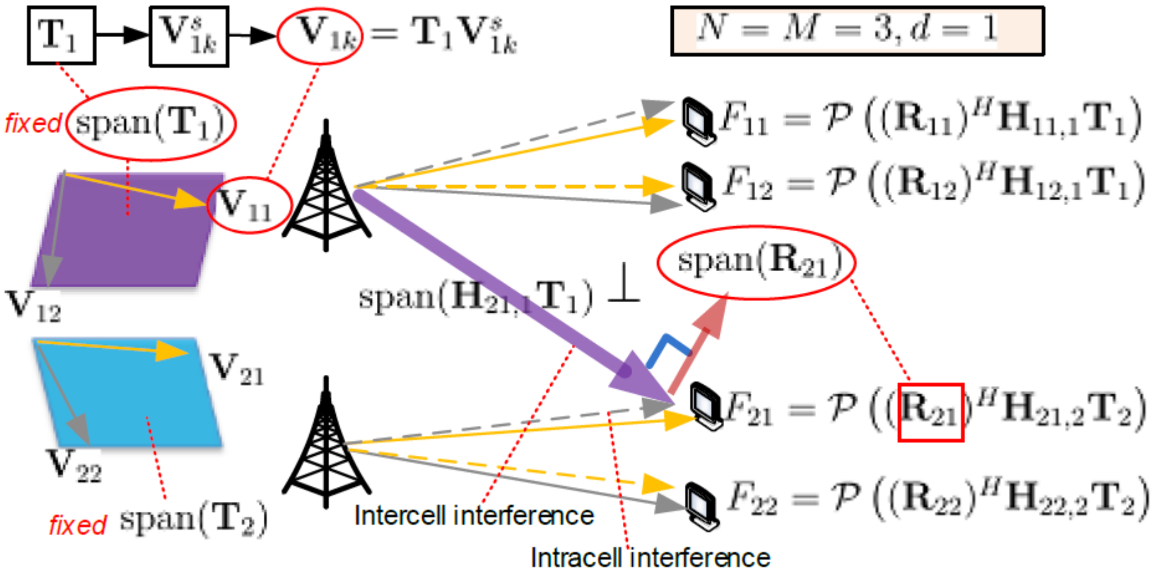
Remark (Backward Compatibility with Conventional Results): If the row space of the concatenated channel matrices of ***all cross links*** are fed back, Corollary 1.1 will be reduced to conventional feasibility result (Theorem 2) in [M. Razaviyanyn, Z. Luo, *et.al.* 2012].

Go back

Backup slides for chapter 2

Example I for IA in Cellular Network

- **Strategy I:** Select a *subset of BSs* to have *fixed outer precoders*



Full CDI	Proposed
64	4

- Insights/Strategy: have a *subset of BSs have fixed outer precoders*. The group of BSs {1 ,…,G} are partitioned into two subsets $\mathbb{B}_g^{I}=\{1,\cdots,g\}$ $\mathbb{B}_g^{II}=\{g+1,\cdots,G\}$

Type-I BSs

Type-II BSs have fixed outer precoder

Example II for IA in Cellular Network

- **Strategy II:** Select a part of the antennas to feedback
  - G = 2 BSs (N = 5), K = 3 MS (M = 3), d = 1,
  - G = 2 BSs (N = 5), K = 3 MS (M' = 2), d = 1 network is already IA feasible. Hence, we can feedback the CSI *sub-matrices.*
  - Insights/Strategy: reduce the CSI feedback antennas. Feedback the left upper $m_{jk} \times n_i$ submatrix of $\mathbf{H}_{jk,i}$.

Full CDI	Proposed
168	108

- Based on the two strategy, we define the CSI feedback strategy as

$$\mathcal{L} = \left\{\{m_{jk} : \forall j,k\}, g, \{n_i : \forall i \in \mathbb{B}_g^I\}\right\} \in \mathbb{L}'$$

Proposed Solution to Q2

Q2: IA feasible conditions under partial CSIT in MIMO cellular network

- Necessary conditions $\mathcal{L} \in \mathbb{L}'_{ne}$ for Problem 2.1

Theorem 2.1 (Necessary Feasibility Conditions). *If Problem 2.1 is feasible, the CSI feedback profile $\mathcal{L}$ should satisfy: 1) $m_{jk} - \sum_{i \in \mathbb{B}_g^{II} \setminus \{j\}} Kd - d \geq 0, \forall j, k$, 2) $N \geq Kd$, $n_i \geq Kd$, $i \in \mathbb{B}_g^I$, 3) $\forall \mathcal{J}_{sub}^{[r]} \subseteq \{(j,k) : \forall j, k\}, \mathcal{J}_{sub}^{[t]} \subseteq \mathbb{B}_g^I$,*

$$\sum_{(j,k) \in \mathcal{J}_{sub}^{[r]}} \left(m_{jk} - \sum_{i \in \mathbb{B}_g^{II} \setminus \{j\}} Kd - d \right) + \sum_{i \in \mathcal{J}_{sub}^{[t]}} K(n_i - Kd) \geq \sum_{j \in \mathcal{J}_{sub}^{[r]}} \sum_{i \in \mathcal{J}_{sub}^{[t]} \setminus \{j\}} Kd. \tag{2.1}$$

- Sufficient conditions $\mathcal{L} \in \mathbb{L}'_{sf}$ for Problem 2.1

Theorem 2.2 (Sufficient Feasibility Conditions). *Suppose $\mathcal{L}$ satisfies the three conditions in Theorem 2.1. If $\mathcal{L}$ further satisfies $d \mid n_i$, $\forall i \in \mathbb{B}_g^I$, or $Kd \mid (m_{jk} - d)$, $\forall j, k$, Problem 2.1 is feasible.*

Performance under limited feedback bits

Suppose a total of B_{tot} CSI feedback bits to quantize and feedback the partial CSI $\{F_{jk} : \forall j, k\}$. The sum average throughput under limited feedback is

$$R_{lim} = \sum_{j=1}^{G}\sum_{k=1}^{K} \mathbb{E}\left\{\log\det\left(\mathbf{I}_d + \frac{P}{Kd}(\hat{\mathbf{U}}_{jk}^H \mathbf{H}_{jk,j}\hat{\mathbf{V}}_{jk})(\hat{\mathbf{U}}_{jk}^H \mathbf{H}_{jk,j}\hat{\mathbf{V}}_{jk})^H\left(\mathbf{I}_d + \mathbf{\Phi}_{jk}\right)^{-1}\right)\right\}$$

Theorem 2.3 (Throughput Bounds). *R_{lim} is bounded by*

$$GKd\int_0^{\infty}\log\left(1+\frac{P}{Kd}\cdot v\right)\cdot f(v)dv \triangleq R_{per} \geq R_{lim} \tag{2.5}$$

$$\geq R_{lb} = R_{per} - \sum_{j=1}^{G}\sum_{k=1}^{K} d\cdot\log\left(1+\frac{P}{d^2}c_{jk}\cdot 2^{-\frac{B}{D(\mathcal{L})}}\right)$$

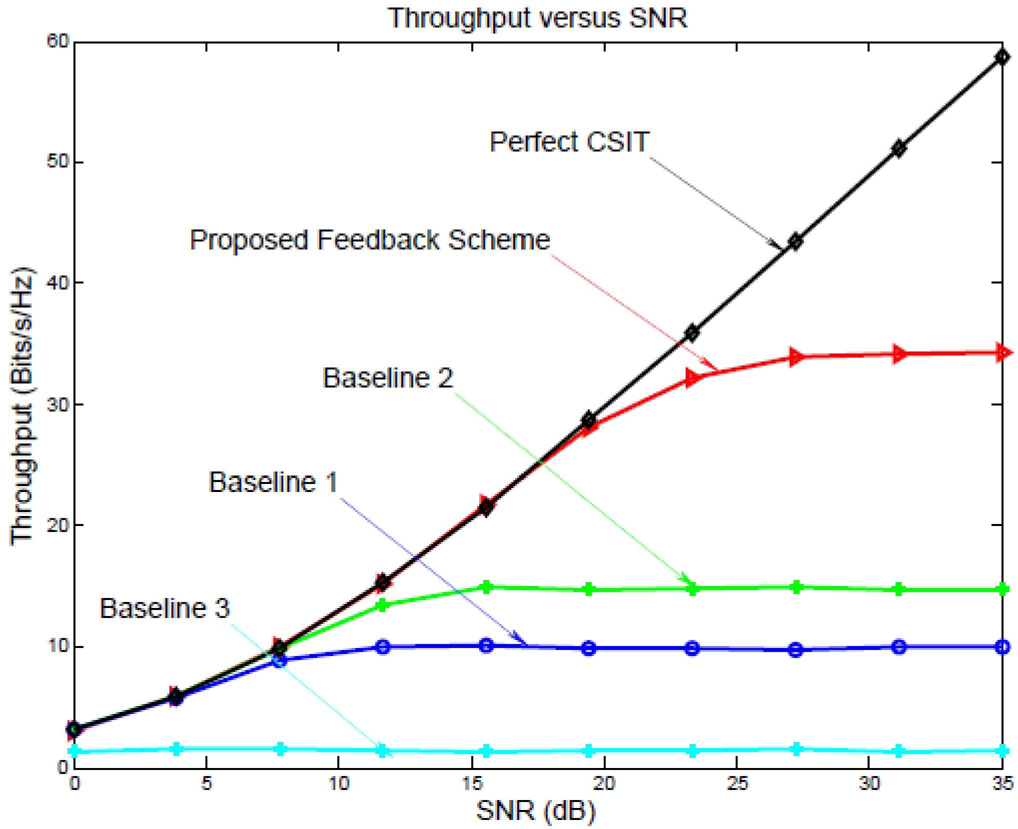
where $f(v)$ is the marginal probability density function of the unordered eigenvalues of the $(d\times d)$ central Wishart matrix with d degrees of freedom and covariance matrix $\mathbf{I}$ [64] (pp 32-33).

Corollary 2.2 (Throughput Scaling). *When the total number of CSI feedback bits B_{tot} is given by:*

$$B_{tot} = D(\mathcal{L})\log P, \tag{2.6}$$

R_{lim} satisfies $\lim_{P\to\infty}\frac{R_{lim}}{\log P} = GKd$.

Simulation Results for MIMO Cellular Networks



Baseline1: full CDI

Baseline2: CSI submatrix feedback.

Baseline3: Random Beamforming

Figure 4.3: Throughput versus transmit SNR under $B_{tot} = 800$ in a $G = 3$, $K = 2$, $N = M = 4$, $d = 1$ network.

Backup slides for chapter 3

CS with Prior Channel Support Infor.

- Convergence speed of the proposed Algorithm 1

Theorem 3.2. *Denote* $\rho = \frac{PT\|\mathbf{H}\|_F^2}{M\|\mathbf{N}\|_F^2}$. *If* $\rho > \left(\frac{C_2+C_1-1}{1-C_1}\right)^2$, *the threshold parameter* γ *in Algorithm 1 satisfies* $\gamma > \frac{C_2\eta}{1-C_1}$, *and* $\Phi = \sqrt{\frac{M}{T}}\mathbf{X}^H$ *satisfies a* s_2*-th order RIP with* $\delta_{s_2} \le 0.153$, *then Step 2 of Algorithm 1 will stop with no more than* n_{co} *iterations where* n_{co} *is given by*

$$n_{co} = \log_{C_1}\left[\frac{\gamma - \frac{C_2\|\mathbf{N}\|_F}{1-C_1}}{\sqrt{1+\delta_{\bar{s}|d}}\rho^{\frac{1}{2}}\|\mathbf{N}\|_F + \|\mathbf{N}\|_F - \frac{C_2\|\mathbf{N}\|_F}{1-C_1}}\right]. \quad (3.2)$$

i.e., converges in $\mathcal{O}(\log \mathrm{SNR})$ steps

Total Complexity $\mathcal{O}\left(\bar{s}^2 T \log \mathrm{SNR}\right)$

Backup slides for chapter 4

Performance Analysis

- Define the following support recovery events:

Θ_c: In J-OMP algorithm, the estimated support Ω_c^e is correct, i.e., $\Omega_c^e \subseteq \Omega_c$.

Θ_i: in J-OMP algorithm, the estimated Ω_i^e is correct, i.e., $\Omega_i^e = \Omega_i$.

- We obtain a bound on the normalized mean absolute error of CSIT as below.

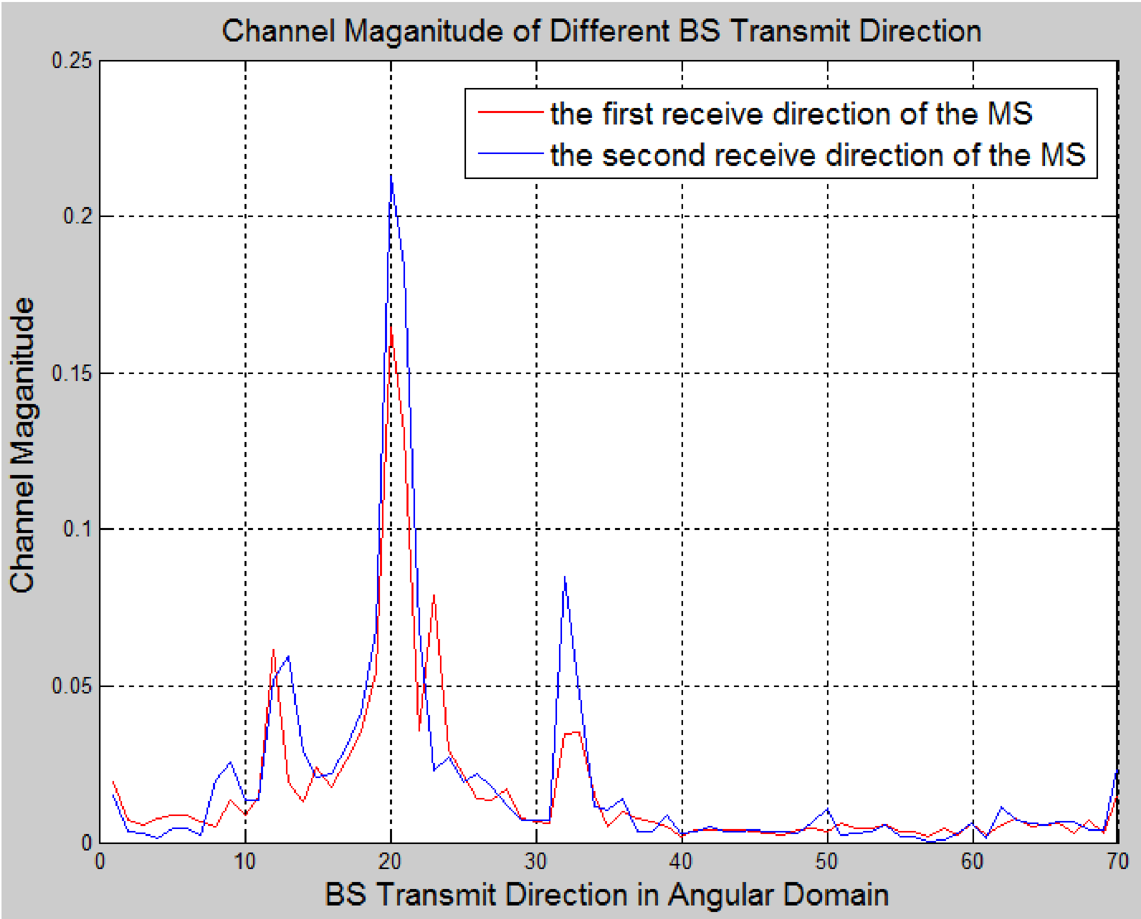
Theorem 3.1 (CSIT Estimation Quality). *The NMAE of* $\mathbf{H}_i$ *is bounded by*

$$\mathbb{E}\left(\frac{\|\mathbf{H}_i - \mathbf{H}_i^e\|_F}{\|\mathbf{H}_i\|_F}\right) \leq \sqrt{\frac{MNs}{PT(1-\delta_s)}}\frac{\Gamma\left(N-\frac{1}{2}\right)}{\Gamma\left(N\right)} + \varepsilon\left(1+\sqrt{\frac{1+\delta_1}{1-\delta_s}}\right) + E_i$$

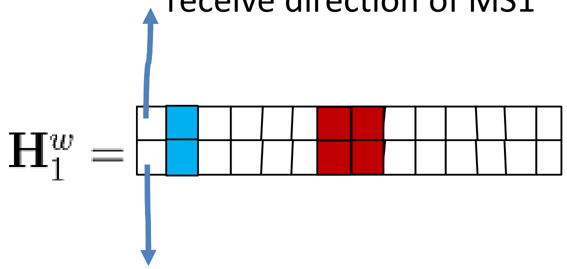
where $E_i = (2 - Pr(\Theta_c \mid \Lambda) - Pr(\Theta_i \mid \Theta_c \Lambda))\left(\frac{1-\delta_s+\delta_{2s}}{1-\delta_s}\right)$, *and* δ_s *and* δ_{2s} *are the s-th and 2s-th restricted isometry constants of* $\bar{\mathbf{X}}$ *respectively.*

Simulations using ITU-R model

- We generate the channel using the ITU-R IMT-Advanced model for Urban Micro Scenario and obtain the following results.



First row corresponds to first receive direction of MS1



Second row corresponds to the second receive direction of MS1

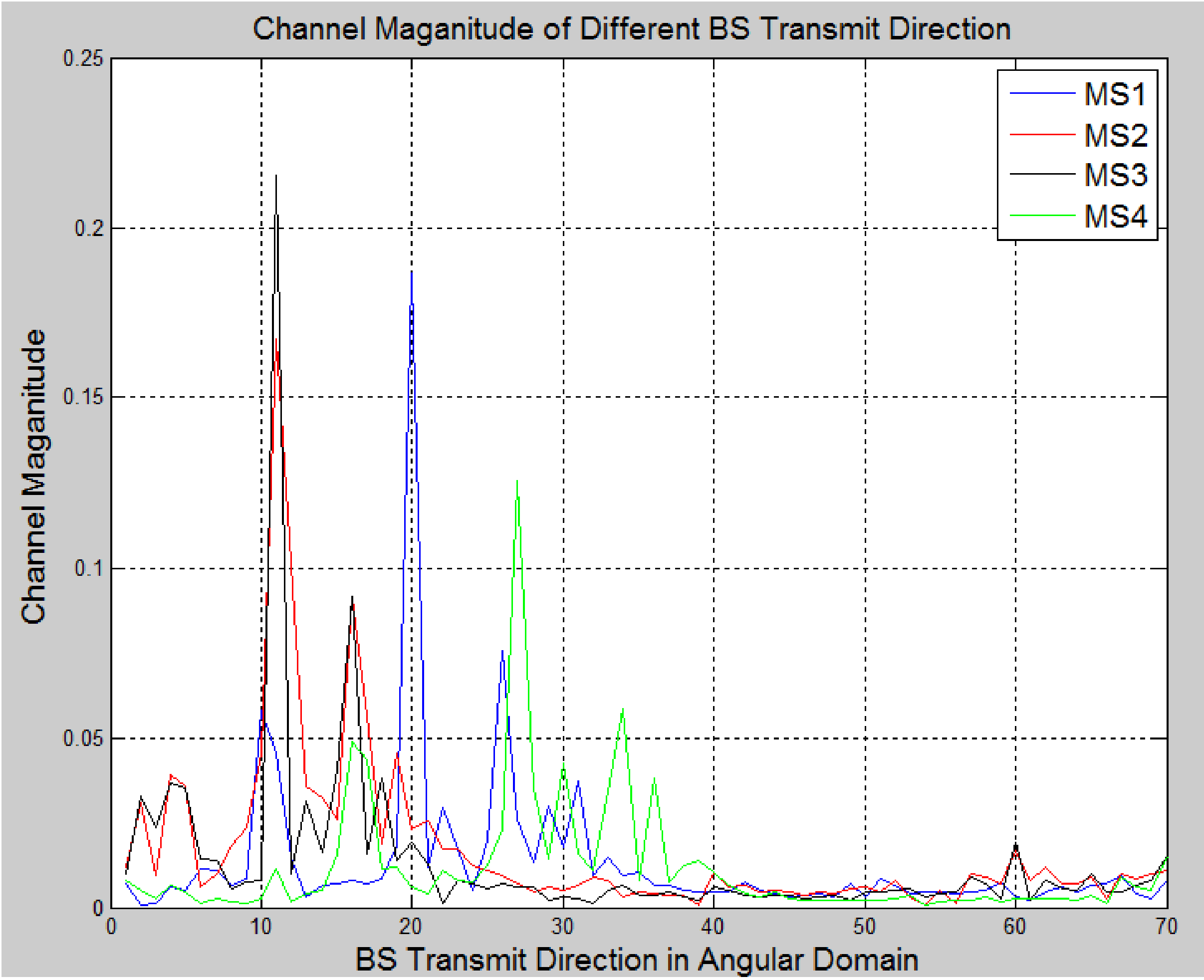
BS: M = 70 antennas

MS: N = 2 antennas

Non-zero coefficients

Zero coefficients

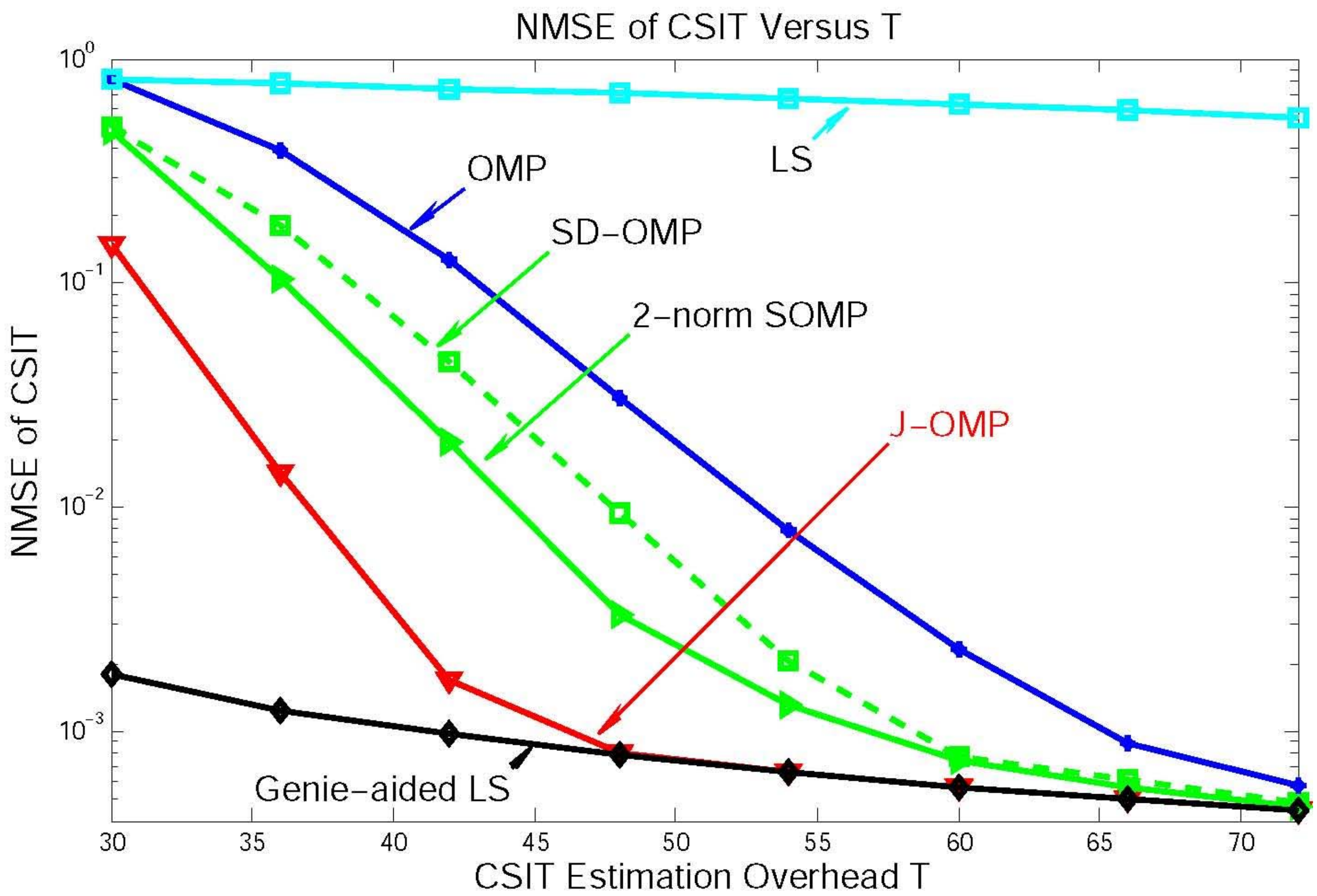
Simulations using ITU-R model



Consider K = 4 MSs, different MSs share some of their channel sparsity support due to the shared common scattering around the BS.

BS: M = 70 antennas
MS: N = 2 antennas

Simulation Results



$$s_c = 9, s = 17, N = 2$$