CSI Acquisition Strategies for Interference Mitigation in Massive MIMO Networks

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Motivation

Channel State Information (CSI) Acquisition
CSI Acquisition is Important to Achieve Interference Mitigation

- **Interference** is a key performance bottleneck of wireless communication

- By adapting the precoder / decorrelator to the channel state information (CSI), many advanced techniques are proposed to mitigate the interference, e.g.,
  - Technique of zero forcing (ZF) [Jindal, *et al.* 2006]
  - Technique of weighted MMSE (WMMSE) [Shi, Luo, *et al.* 2011]
  - Technique of Interference alignment (IA) [Cadambe, Jafar, *et al.* 2008]
Performance Gain of Interference Mitigation with CSI

• These interference mitigation schemes achieve dramatic performance gains:

![Diagram showing interference mitigation schemes](image)
Acquiring CSI is **Costly**

• However, these interference mitigation techniques require CSI and acquiring CSI at the receiver (CSIR) or at the transmitter (CSIT) is costly, especially for CSIT acquisition
  – E.g. how to acquire CSIT in FDD systems
    • Phase I: Transmitter (Tx) sends pilot training symbols to probe the channel on the forward channel
      ![Phase I Diagram]
    • Phase II: Receiver (Rx) estimates and feedbacks the channel information to Tx side
      ![Phase II Diagram]
Implication

• Take CSI acquisition cost into consideration in the interference mitigation designs
  ✓ Reduce the CSI acquisition cost
  ✓ Tradeoff analysis between the interference mitigation performance and the CSI acquisition cost.
Overview

Outline of My Thesis Research
Research Outline

CSI feedback Reduction for IA
- in interference network with IA processing
  [Rao, Ruan, Lau, TSP 13]
- in cellular network with IA processing
  [Rao, Lau, TSP 14]

CSI Acquisition Strategies

CSI Acquisition Design
- Downlink CSI (including both CSI training and feedback)
  [Rao, Lau, TSP 14]
- Uplink CSI (future work)
  [Rao, Lau, ??]
Background

CSI Feedback Reduction for IA
Background of IA

• Technique of interference alignment (IA)
  – First proposed by in [Maddah-Ali, et. al. 2008]
  
  – **Basic idea**: Align the interference from different Txs into a lower dimensional subspace at the Rx

  – **Performance advantage**: Achieve optimal capacity scaling law w.r.t. SNR [Cadambe, Jafar, et.al. 2008]
IA in MIMO interference network

• Network Topology
  1: $K$-user MIMO interference network
  2: The $i$-th Tx and Rx have $N_i$ and $M_i$ antennas respectively
  3: $d_i$ data streams are demanded for the $i$-th Tx-Rx pair.
  4: Denote the channel from the $i$-th Tx to $j$-th Rx as $H_{ji} \in \mathbb{C}^{M_j \times N_i}$.

• Signal Model
  The received signal $y_j \in \mathbb{C}^{d_j \times 1}$ at the $j$-th Rx is given by
  \[
  y_j = U_j^H \left( \sum_{i=1}^{K} H_{ji} V_i x_i + n_j \right)
  \]

• Problem Formulation
  **Problem 1.1** (Conventional IA). Find the set of precoders $\{ V_i \in \mathbb{C}^{N_i \times d_i} : \forall i \}$
  and decorrelator $\{ U_j \in \mathbb{C}^{M_j \times d_j} : \forall j \}$, such that:
  \[
  \text{rank}(U_j^H H_{jj} V_j) = d_j, \forall j, \tag{1.1}
  \]
  \[
  U_j^H H_{ji} V_i = 0, \forall i, j, i \neq j \tag{1.2}
  \]
Conventional IA Results

• When Problem 1.1 feasible, \[ C = \left( \sum_{k=1}^{K} d_k \right) \log_2(\text{SNR}) + o(\log_2(\text{SNR})) \]

• There are two questions associated with Problem 1.1.
  – **Feasibility conditions**: when will Problem 1.1 has solution?
    (answer: feasibility study in [M. Razaviyayn, Z. Luo, et.al. 2012])
  – **Transceiver design**: Given that Problem 1.1 is feasible, how to find the solution?
    (answer: AILM in [Gomadam, Jafar, et.al. 2011])

• **Limitations** of these IA designs: at least full channel direction information (CDI) is required.

Consider a \( K = 6 \) MIMO interference network where \( M = 12, N = 36 \) and \( d_i = d, \forall i \).

Then from conventional IA study under full CDI, \( d \leq 6 \)
Question: For a given data stream requirements $\{d_i : \forall i\}$ in the network, are there any approaches to reduce the CSI feedback overhead while still achieving IA?
Background of CSI Feedback Reduction

- Note that we focus on *CSI filtering*\(^1\) only, which is different from CSI quantization ([Rajesh T. K. *et al.* 2010]).

\[ F_jk(H) \]

E.g., for a \((2 \times 3)\) matrix \(H\),

1. \( F(H) = H(1,:) \)

2. \( F(H) = \{v : Hv = 0\} \)

---

\(^1\) In CSI filtering, only those parts of CSI that are essential to IA design are *selected* to be fed back.
Literature Works

• CSI feedback reduction for IA (focusing on CSI filtering)

<table>
<thead>
<tr>
<th>Works</th>
<th>CSI feedback reduction</th>
<th>IA Design (Feasibility Conditions &amp; Transceiver Algorithm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[Razaviyan, et.al. 2012]</td>
<td>Full CSIT</td>
<td>Conventional</td>
</tr>
<tr>
<td>[Jafar, et.al. 2011]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[K. R.T., et.al. 2010]</td>
<td>Full CDI feedback</td>
<td>Same as above</td>
</tr>
<tr>
<td>[Rezacee, et.al., 2013]</td>
<td>CSI submatrix feedback</td>
<td>Direct extension from above</td>
</tr>
<tr>
<td>This work</td>
<td>more holistic set of</td>
<td>Different feasibility conditions and transceiver algorithms</td>
</tr>
<tr>
<td></td>
<td>CSI reduction strategies</td>
<td></td>
</tr>
</tbody>
</table>

Table 1.2: A short summary of previous works on IA with partial CSIT.
System Model

• CSI Feedback Topology
  1: The Rx $j$ has perfect local cross-link CSI $\mathcal{H}_j = (\cdots, H_{ji}, \cdots)_{i \neq j}$.
  2: The Rx $j$ feedbacks partial CSI $\mathcal{H}_j^{fed} = F_j(\mathcal{H}_j)$.
  3: The feedback CSI can be shared among the Txs.

• CSI Feedback Function $\mathcal{H}_j^{fed} = F_j(\mathcal{H}_j)$
  The CSI feedback function at the $j$-th Rx $F_j$
  
  $F_j : \prod_{i=1}^{K} \mathbb{C}^{M_j \times N_i} \rightarrow \prod_{i=1}^{k_j} \mathbb{G}(A_{ji}, B_{ji})$

• Example

  The Grassmann manifold $\mathbb{G}(A, B)$, $(A \leq B)$, is the set of all $A$-dimensional linear subspaces in $\mathbb{C}^{B \times 1}$ $(A \leq B)$.

  If $U^H H V = 0$, then we have $U^H (aH) V = 0, \forall a \in \mathbb{C}$. Hence, it is sufficient to feedback the CDI of $H \in \mathbb{C}^{N \times M}$ for IA, i.e., $F(H) = \{aH : a \in \mathbb{C}\} \in \mathbb{G}(1, MN)$
CSI Feedback Cost

• How to quantify the CSI feedback cost associated with \( \{ F_j(\mathcal{H}_j) \} \)

  Define the CSI feedback cost as the sum dimension \( D \) of the Grassmann manifolds \( \{ \mathcal{G}(A_{ji}, B_{ji}) : \forall i, j \} \) that contain the CSI feedback back \( \{ F_j(\mathcal{H}_j) \} \), i.e.,

  \[
  D = \sum_{j=1}^{K} \sum_{i=1}^{k_j} A_{ji}(B_{ji} - A_{ji}).
  \]

• First order metric of CSI feedback dimension to measure the CSI feedback cost

  e.g., to keep a constant quantization distortion \( \Delta \), the quantization bits \( B \) has to scale with the dimension \( D \) as \( B = \mathcal{O}(D \cdot \log \frac{1}{\Delta}) \).
Problem Formulation

• **Problem 1.2** (IA under Partial CSIT). Find \( \{V_j : \forall j\} \) and \( \{U_j : \forall j\} \), such that:

\[
\begin{align*}
\text{rank}(U_j^H H_{jj} V_j) &= d_j, \quad \forall j, \quad (1.3) \\
U_j^H H_{ji} V_i &= 0, \quad \forall i, j, i \neq j \quad (1.4)
\end{align*}
\]

\( \{V_i : \forall i\} \) can only be adaptive to the partial CSI feedback \( \{F_j(H_j) : \forall j\} \quad (1.5) \)

• Three key questions associated with Problem 1.2

Q1: How to reduce CSI feedback for IA?, i.e., what is \( \{F_i\} \)

Q2: IA feasible conditions

\[
C = \left( \sum_{k=1}^{K} d_k \right) \log_2(\text{SNR}) + o(\log_2(\text{SNR}))
\]

Q3: Find transceiver solution under partial CSIT
Proposed Solution to Q1

Q1: How to reduce CSI feedback for IA?, i.e., what is \{F_j\}

- **Toy Example I**
  
  \[ H_{31} V_1 = 0 \]
  \[ H_{12} V_2 = 0 \]
  \[ H_{23} V_3 = 0 \]

- **Insights:**
  
  - **Strategy I:** No feedback for a subset of cross links
  - **Strategy II:** Feedback of null space for a subset of cross links.

<table>
<thead>
<tr>
<th>Full CDI</th>
<th>Proposed</th>
</tr>
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<tbody>
<tr>
<td>138</td>
<td>24</td>
</tr>
</tbody>
</table>

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Proposed Solution to Q1

Q1: How to reduce CSI feedback for IA?, i.e., what is \( \{F_j\} \)

- Toy Example II: Strategy III
- Toy Example III: Strategy IV
Proposed Solution to Q1

Q1: How to reduce CSI feedback for IA?, i.e., what is \( \{F_j\} \)

- CSI feedback scheme (characterized by \( \mathcal{L} \))

  Define the CSI feedback profile as

  \[ \mathcal{L} = \{ \{M_i^s, N_i^s : \forall i\}, \{\Omega_j^I, \Omega_j^II, \Omega_j^III, \Omega_j^IV : \forall j\}\} \]

  1. \( \{M_i^s, N_i^s\} \): controls the size of the CSI submatrices to feedback.
  2. \( \{\Omega_j^m : \forall m\} \): partitioning of the cross links w.r.t. the four feedback strategies.

  - There is a 1-1 correspondence between the feedback profile \( \mathcal{L} \) and feedback function \( \{F_j\} \)

- The associated feedback cost is given by (a function of \( \mathcal{L} \))

  \[ D(\mathcal{L}) = \sum_{j=1}^{K} M_j^e \left( \sum_{i \in \Omega_j^{IV}} N_i^s - M_j^e \right)^+ + \sum_{j=1}^{K} \sum_{i \in \Omega_j^{III}} M_j^e (N_i^s - M_j^e). \]
Proposed Solution to Q2

Proposed Solution to Q2

Q2: IA feasible conditions under partial CSIT

• Challenge: Hidden CSI knowledge constraint in Problem 1.2 (the precoders are only adaptive to partial CSIT feedback).

• We obtain sufficient conditions on the CSI feedback profile $\mathcal{L}$ to ensure that Problem 1.2 has solutions.

Theorem 1.1 (Sufficient Feasibility Conditions). If the feedback profile $\mathcal{L}$ satisfies the following conditions:

1) $N_i^e = N_i^g - \sum_{j:i \in \Omega_j} M_j^e \geq d_i$, $M_j^e = M_j^g - \sum_{i \in \Omega_j} N_i^e \geq d_i^0 \ \forall i$,

2) the row vectors of all the matrices $\{X_{ji} : \forall j, i \in \Omega_j^I\}$ are linearly independent, then

Problem 1.2 (Proposed IA under Partial CSIT) is feasible almost surely, where

$$X_{ji} = \begin{bmatrix} 0 & (G_{ji}^{(2)})^T \otimes I_{d_j^0} & 0 & I_{d_i} \otimes G_{ji}^{(3)} & 0 \end{bmatrix}$$

(1.10)

and $\overline{M} = \sum_{i=1}^K (d_i^0 (M_i^e - d_i^0) + d_i (N_i^e - d_i))$, $n_{ji} = \sum_{p=j+1}^K d_p^0 (M_p^e - d_p^0) + \sum_{q=1}^{i-1} d_q (N_q^e - d_q)$, $m_{ji} = \sum_{p=1}^{j-1} d_p^0 (M_p^e - d_p^0)$, $k_{ji} = \sum_{q=i+1}^K d_q (N_q^e - d_q)$, $d_j^0 = d_j + \sum_{i \in \Omega_j} d_i$. 

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Proposed Solution to Q2

Q2: IA feasible conditions under partial CSIT

• Explicit sufficient conditions on $\mathcal{L}$ under divisible cases.

Corollary 1.1 (Explicit Sufficient Feasibility Conditions). Suppose $d_i = d, \forall i$. If the feedback profile $\mathcal{L}$ satisfies:
1) $d | M_i^s$, $d | N_i^s, \forall i$,
2) $N_i^e = N_i^s - \sum_{j: i \in \Omega_j^1} M_j^e \geq d_i$, $M_j^s - \sum_{i \in \Omega_j^1} N_i^s \geq d_j^0, \forall i$,
3) Denote $V_i = d_i(N_i^e - d_i), \forall i$, $U_j = d_j^0(M_j^e - d_j^0), \forall j$; $C_{ji} = d_j^0d_i$, and $V_i, U_j$ and $C_{ji}$ satisfy

$$\sum_{j: (j,i) \in \Omega_{sub}} U_i + \sum_{i: (j,i) \in \Omega_{sub}} V_i \geq \sum_{j,i: (j,i) \in \Omega_{sub}} C_{ji}, \forall \Omega_{sub} \subseteq \{(j,i): \forall j, i \in \Omega_j^IV\}. \quad (1.11)$$

then Problem 1.2 (Proposed IA under Partial CSIT) is feasible almost surely.

Remark (Backward Compatibility with Conventional Results): If the row space of the concatenated channel matrices of all cross links are fed back, Corollary 1.1 will be reduced to conventional feasibility result (Theorem 2) in [M. Razaviyany, Z. Luo, et.al. 2012].
Proposed Solution to Q3

Q3: Find transceiver solution under partial CSIT

• We propose a novel transceiver algorithm under the proposed CSI feedback scheme

The proposed modified AILM solves the following two problems ($\mathcal{P}_1$, $\mathcal{P}_2$) alternatively until convergence.

$$\mathcal{P}_1 : \min_{\{\mathbf{u}^b_j : (\mathbf{u}^b_j)^H \mathbf{u}^b_j = \mathbf{I}_{d_j}, \forall j\}} I = \sum_{j,i : i \in \Omega^j_V} \text{tr} \left( ((\mathbf{u}^b_j)^H \mathbf{G}_{ji} \mathbf{v}^a_i) ((\mathbf{u}^b_j)^H \mathbf{G}_{ji} \mathbf{v}^a_i)^H \right).$$

$$\mathcal{P}_2 : \min_{\{\mathbf{v}^a_i \in \mathbb{C}^{N_i^a \times d_i} : (\mathbf{v}^a_j)^H \mathbf{v}^a_j = \mathbf{I}_{d_j}, \forall i\}} I = \sum_{j,i : i \in \Omega^j_V} \text{tr} \left( ((\mathbf{u}^b_j)^H \mathbf{G}_{ji} \mathbf{v}^a_i) ((\mathbf{u}^b_j)^H \mathbf{G}_{ji} \mathbf{v}^a_i)^H \right).$$

With the converged solution $\{\mathbf{v}^a_i, \mathbf{u}^b_j\}$, the overall solution to Problem 1.2 is given by

$$\mathbf{v}_i = \begin{bmatrix} \mathbf{S}_i^T \mathbf{v}^a_i \\ \mathbf{0} \end{bmatrix}, \quad \mathbf{u}_j = v_{d_j} \left( \sum_{i \neq j} (\mathbf{H}_{ji} \mathbf{v}_i) (\mathbf{H}_{ji} \mathbf{v}_i)^H \right), \forall i, j$$
A short summary of the results

- Problem of IA under Partial CSIT

Q1: How to reduce CSI feedback for IA?

Q2: IA feasible condition under partial CSIT

Q3: IA transceiver design under partial CSIT

A1: We propose a set of CSI feedback reduction strategies

A2: We establish sufficient conditions to make IA feasible under the proposed partial CSI feedback scheme

A3: We extend conventional IA transceiver designs (i.e., AILM algorithm) to the cases with only partial CSIT
CSI Feedback Profile Design $\mathcal{L}$

- Problem formulation

The problem of reducing the CSI feedback overhead subject to a given data streams requirement for the Tx-Rx pairs $\{d_i : \forall i\}$, can be formulated as:

**Problem 1.3 (CSI Feedback Design).**

$$\min_{\mathcal{L}} D(\mathcal{L}) \quad \text{s.t.} \quad \mathcal{L} \text{ satisfies the sufficient IA feasibility conditions in Theorem 1.1.}$$

1: Problem 1.3 is very challenging due the combinatorial nature of CSI feedback profile designs ($\mathcal{L}$).

2: We propose a low-complexity greedy algorithm to derive solution.
Theorem 1.2 (Performance-Cost Tradeoff on a Symmetric MIMO Interference Network).
Consider a $K$-user MIMO interference network where $d_i = d$, $M_i = M$, $N_i = N$, $\forall i$ and $M$, $N$, $d$ satisfy $2 \mid M$, $M \leq 2K + 1$, $N = \frac{1}{2}KM$, $d \mid M$. The tradeoff between the data stream $d$ and the feedback dimension $D_p$ is summarized below:

<table>
<thead>
<tr>
<th>Data Stream $d$</th>
<th>Feedback dimension $D_p$</th>
<th>full CDI $D_{full}$</th>
<th>Submatrix Feedback $D_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d \leq \frac{M}{K}$, $d \mid M$</td>
<td>0</td>
<td>$K(K - 1)$</td>
<td>$K(K - 1) \cdot (M \times ((K + 1)d - M) - 1)^+$</td>
</tr>
</tbody>
</table>
| $d = \frac{M}{K - \kappa}$, $1 \leq \kappa \leq K - 2$ | $(K + 1)d^2 - Md$ | $(\frac{1}{2}KM^2 - 1)$ | \[
(K - 1)^2 \]

Consider a $K = 6$ MIMO interference network where $M = 12$, $N = 36$ and $d_i = d$, $\forall i$.

Then from conventional IA feasibility study under full CDI, $d \leq 6$
Simulation Results

Baseline 1: full CDI

Baseline 2: Truncated feedback

Baseline 3: CSI submatrix feedback

Figure 3.4: Throughput versus SNR under a $K = 4$, $[N_1, \ldots N_4] = [5, 4, 4, 3]$, $[M_1, \ldots M_4] = [4, 3, 2, 4]$, $[d_1, \ldots d_4] = [2, 1, 1, 1]$ MIMO interference network and the sum feedback bit constraint is 400.
(Thesis Chapter 4) Extension to

MIMO Cellular Networks
System Model

• MIMO Cellular Network
  – G base stations (BSs) and each BS has N antennas.
  – Each BS has K mobile station (MS) and each MS has M antennas.
  – d data streams are transmitted from the BS to each MS (Given DoF requirement).

• CSI Feedback Topology
  1. The (j, k)-th MS has perfect local CSI \( \mathcal{H}_{jk} = (H_{jk,1}, H_{jk,2}, \ldots H_{jk,G}) \)
  2. The (j, k)-th MS feedbacks partial CSI \( \mathcal{H}_{jk}^{fed} = F_{jk}(\mathcal{H}_{jk}) \)
  3. The feedback CSI can be shared among the BSs \( \{1, \ldots, G\} \).

• Challenge
  There are both intra-cell and inter-cell interference in cellular networks, which makes the dependencies of IA precoders on CSIs, very complicated.
Problem Formulation

• Two Stage Precoding at BS

The precoder for the \((j, k)\)-th MS is given by

\[
V_{jk} = T_j V_{jk}^s
\]

\(T_j \in \mathbb{C}^{N \times K_d}\): Outer precoder at \(j\)-th BS.

\(V_{jk}^s \in \mathbb{C}^{K_d \times d}\): Inner precoder for the \((j, k)\)-th MS.

• Problem formulation

**Problem 2.1 (IA under Partial CSIT).** Find \(\{T_j, V_{jk}^s\}\) and \(\{U_{jk}\}\), such that

\[
\text{rank}(U_{jk}^H H_{jk,j} T_j V_{jk}^s) = d, \forall j, k;
\]

\[
U_{jk}^H H_{jk,j} T_j V_{jp}^s = 0, \forall j, k \neq p; \quad \text{(intracell IA constraints)}
\]

\[
U_{jk}^H H_{jk,i} T_i = 0, \forall j, k, i \neq j; \quad \text{(intercell IA constraints)}
\]

\(\{T_j, V_{jk}^s : \forall j, k\}\) can only be adaptive to partial CSI \(\{F_{jk}(\mathcal{H}_{jk}) : \forall j, k\}\).

\(\mathbb{C}^N\times\mathbb{R}^d\)

IA constraints

CSI knowledge constraint
Proposed Scheme

• Regarding Problem 2.1, we shall answer the following questions:

Q1: How to reduce CSI feedback for IA?, i.e., what is \( F_{jk} \)

A1: CSI feedback profile \( \mathcal{L} \) with CSI feedback cost \( D(\mathcal{L}) \)

Q2: IA feasible conditions under partial CSIT

A2: Necessary Conditions
Sufficient Conditions

Q3: How to find transceiver solution under partial CSIT

A3: A novel transceiver algorithm
----Extended from AILM
----Adapt to partial CSI in MIMO cellular network
CSI Feedback Profile Design $\mathcal{L}$

- Problem formulation
  - The problem of reducing the CSI feedback cost subject to a given $d$ data streams requirement for each MS, can be formulated as:

  **Problem 2.2 (CSI Feedback Design).**

  $$\min_{\mathcal{L}} D(\mathcal{L})$$

  s.t. Problem 2.1 is feasible under $\mathcal{L}$.  \hspace{1cm} (2.2)

  - We replace (2.2) with its sufficient conditions $\mathcal{L} \in \mathbb{L}_{s_f}$, and propose a low complexity *achievable* solution $\mathcal{L}_0$ that satisfies the sufficient conditions

    $$D(\mathcal{L}^\ast) \leq D(\mathcal{L}_0)$$
Problem 2.2 (CSI Feedback Design).

\[
\min_{\mathcal{L}} \quad D(\mathcal{L}) \\
\text{s.t.} \quad \text{Problem 2.1 is feasible under } \mathcal{L}. \\
\mathcal{L} \in \mathbb{L}_{ne} \quad \text{(Nece. cond)} \\
\text{Lower Bound}
\] (2.2)

– We replace (2.2) with its necessary conditions \( \mathcal{L} \in \mathbb{L}_{ne} \) and derive an lower bound on the optimal feedback dimension \( D_{\text{low}} \leq D(\mathcal{L}^*) \)

\[
D_{\text{low}} \leq D(\mathcal{L}^*) \leq D(\mathcal{L}_0)
\]

– We show that \( D_{\text{low}} \to D(\mathcal{L}_0) \) as \( G \to \infty \), and hence the proposed solution \( \mathcal{L}_0 \) is asymptotically optimal.

Corollary 2.1 (Asymptotic Optimality of \( \mathcal{L}_0 \)). Suppose the number of antennas \( N, M \) are given by \( N = [C_1 K G], M = [C_2 K G] \), where \( 0 < C_1, C_2 < d, d < C_1 + C_2 \). As \( G \to \infty \), we have

\[
\lim_{G \to \infty} \frac{D(\mathcal{L}_{\text{low}})}{G^4 K^3} = \lim_{G \to \infty} \frac{D(\mathcal{L}_0)}{G^4 K^3} = \frac{(d - C_1)(d - C_2)^2}{C_1}.
\]
Summary

- We extend the framework to achieve CSI feedback reduction for IA from MIMO interference networks to MIMO cellular networks.

- The framework consists of the following components:
  - A set of CSI feedback reduction strategies for MIMO cellular network with IA processing
  - IA feasibility study under partial CSIT
  - IA transceiver algorithm under partial CSIT

- We post the problem of CSI feedback cost minimization subject to a given DoFs requirement, and we propose a low-complexity asymptotical optimal solution.
From MIMO to Massive MIMO

CSIT Acquisition in FDD Massive MIMO
Related Literature Works

• Group I: Conventional (LS or MMSE) based CSIT training and feedback ([Biguesh, et.al. 2006]):
  – Require that the training and feedback overhead scales with the number of antennas at the BS, $O(M)$.
    • (a): The BS sends a sequence of training pilots $X$;
    • (b): The $i$-th MS observes $Y_i$, and estimate $H_i$ via LS.
    • (c) The MS feedbacks estimated $H_i$ to the BS side.

$$Y_i = H_iX + N_i \quad \Rightarrow \quad \hat{H}_i = Y_iX^\dagger$$

• Group II: Recent works of compressive sensing (CS) based schemes, i.e., replace the LS with CS-sparse recovery ([Bajwa, et.al. 2010])
  – They consider to point-to-point scenarios.
• *Multi-user* massive MIMO channels are **jointly sparse**
  
  – **Sparsity due to** limited local scatters at BS (e.g., measurement report [Sayeed, *et.al.* 2006])
  
  – Structured sparsity (joint) due to some physical features:
    • Rich local scatterers at MS
    • Shared local scatterers between different MS (e.g., measurement report [Hoydis, *et.al.*, 2008])

**Target**

*Exploit not only the sparsity but also the joint sparsity in the channels to further enhance the CSI training and feedback efficiency!*
Proposed Scheme in Multi-user Massive MIMO
System Model

• Network Topology
  – 1 BS and K MSs where BS and MS have M antennas (M is large) N antennas.

• Channel Model

(a) Individual joint sparsity due to local scattering at the BS: Denote \( h_{ij} \) as the \( j \)-th row vector of \( \mathbf{H}_i^{\mu} \); then \( \{ h_{ij} : \forall j \} \) are simultaneously sparse, i.e., there exists an index set \( \Omega_i \), \( 0 < |\Omega_i| \ll M, \forall i \), such that

\[
\text{supp}(h_{i1}) = \text{supp}(h_{i2}) = \cdots = \text{supp}(h_{iN}) \triangleq \Omega_i. \tag{4}
\]

(b) Distributed joint sparsity due to common scattering at the BS: Different \( \{ \mathbf{H}_i^{\mu} : \forall i \} \) share a common support\(^4\), i.e., there exists an index set \( \Omega_c \) such that

\[
\bigcap_{i=1}^{K} \Omega_i = \Omega_c. \tag{5}
\]

• Assumption of Statistical Sparsity Information

There exists statistical bound \( S = \{ s_c, \{ s_i : \forall i \} \} \) \((s_c, s_i \ll M)\) available at the BS where \( \Pr(A) > 1 - \varepsilon \) for some small \( \varepsilon \) and \( A \) denotes \( |\Omega_c| \geq s_c, |\Omega_i| \leq s_i, \forall i. \)
1) Observation: the pilot training and feedback overheads both scale as the training length $O(T)$.

2) It is very desirable to reduce $T$ by exploiting the underlying joint channel sparsities.
Question: How to jointly recover the channel based on compressed measurements \( \{Y_i\} \)?

\[
Y_i = H_i X + N_i, \quad \forall i \quad \xrightarrow{\text{CSIT Recovery}} \quad H_i^c, \quad \forall i
\]
Proposed Joint CSIT Recovery

• Proposed Joint-OMP Algorithm
  – **Extended** from conventional OMP and can exploit the joint sparsity structures in the user channel matrices to enhance the CSIT estimation performance.
  
  – **Algorithm flow**

```
<table>
<thead>
<tr>
<th>Common support identification</th>
<th>Individual support identification</th>
<th>LS estimation based on estimated support</th>
</tr>
</thead>
</table>
```

  – **Complexity analysis**: Consider $s_i = s, \forall i$ for simplicity, overall complexity of the J-OMP is $O(KMNST)$
  
  – **Information requirements**: Only the channel sparsity statistics $S = \{s_c, \{s_i : \forall i\}\}, (s_c, s_i \ll M)$ is required.
Question: What is the performance of the proposed CSIT estimation scheme

E.g., \( \mathbb{E} \left( \frac{\|H_i - H_i^e\|_F}{\|H_i\|_F} \right) \)?
Performance Analysis

• Define the following support recovery events:

  $\Theta_c$: In J-OMP algorithm, the estimated support $\Omega_c^e$ is correct, i.e., $\Omega_c^e \subseteq \Omega_c$.

  $\Theta_i$: in J-OMP algorithm, the estimated $\Omega_i^e$ is correct, i.e., $\Omega_i^e = \Omega_i$.

• We obtain a bound on the normalized mean absolute error of CSIT as below.

\[
\begin{align*}
\mathbb{E} \left( \frac{|| H_i - H_i^e ||_F}{|| H_i ||_F} \right) & \leq \sqrt{ \frac{M N s}{P T (1 - \delta_s)} } \left( \frac{(N - \frac{1}{2})}{\Gamma(N)} \right) \\
& \quad + \varepsilon \left( 1 + \sqrt{\frac{1 + \delta_1}{1 - \delta_s}} \right) + E_i
\end{align*}
\]

where $E_i = \left( 2 - Pr(\Theta_c | \Lambda) - Pr(\Theta_i | \Theta_c \Lambda) \right) \left( \frac{1 - \delta_s + \delta_{2s}}{1 - \delta_s} \right)$, and $\delta_s$ and $\delta_{2s}$ are the $s$-th and 2s-th restricted isometry constants of $\bar{X}$ respectively.
Corollary 5.3 (CSIT Quality w.r.t. $\Omega_c$). Suppose $\varepsilon = 0$ in (5.2.6). Scale the threshold parameter $\eta_2$ in Algorithm 5.2 as $\eta_2 = \sqrt{P}$ and let the transmit SNR $P \to \infty$, the number of users $K \to \infty$. If (5.3.8) holds and $p$ in (5.3.9) satisfies $p < \frac{1}{2}(1 - \gamma)$, we have

$$\mathbb{E}\left(\frac{\|H_i - H_i^c\|_F}{\|H_i\|_F}\right) \leq \left(\sum_{t=s_c}^{s} \binom{s}{t} - 1\right) E,$$

(5.4.8)

where $E = \left(\frac{1 - \delta_s + \delta_{2s}}{1 - \delta_s}\right) \times \left(\exp\left(-N\left(\ln \theta - 1 + \frac{1}{\theta}\right)\right) + M \cdot \exp\left(-N\left(\theta - 1 - \ln \theta\right)\right)\right)\).
Simulation Results

NMSE of CSIT Versus T

- OMP
- LS
- SD-OMP
- 2-norm SOMP
- J-OMP
- Genie-aided LS

$s_c = 9$, $s = 17$, $N = 2$
Summary

• We consider CSIT acquisition (includes both CSI training and feedback) in multi-user FDD massive MIMO systems.

• We propose an CS-enabled CSI acquisition scheme to exploit the joint sparsity in the user channel matrices.
  – A {CSIT measurement and feedback} scheme in which user measures the channel locally and feedback the measurements to the BS side for joint CSIT recovery.
  – A joint-OMP algorithm to conduct the CSIT recovery at the BS.

• We obtain simple analytical results showing that the joint channel sparsity structures can be exploited to enhance the CSIT estimation performance.
Future works

CSIR Acquisition in Uplink Massive MIMO
System Model

• Network Topology
  – 1 BS and K MSs where BS and MS have M antennas N antennas respectively. Suppose that both M and K are large.

• Model of Sparse Active Users
  A. Sparse user due to random access or bursty transmissions of user packets.
  B. Temporary correlations between active user sets due to bursty features of user packets.

• Mission: Uplink CSIR Estimation
  A. The MSs transmit a sequence of uplink pilots.
  B. The BS estimate the uplink CSI based on the channel observations.

Compressive uplink pilots

Joint Recovery of \( \{H_i, \forall i \in A\} \)

Active users

inactive user
Publications

• Journal

• Conference
Thank you!

(Any questions?)